# Accelerated Learning Program

## Mathematic 10th (Science Group)

## Unit #1 :- Quadratic Equations

### Exercise # 1.1 :-

Q#1 (i , iii , iv)
Q#2 (ii , iv , v)
Q#3 (i, v , ix)



**Question # 1 (i)** Write the following quadratic equation in the standard form and point out pure quadratic eq.

$$(x+7)(x-3) = -7$$

Sol//

$$(x+7)(x-3) = -7$$

$$x^2 - 3x + 7x - 21 = -7$$

$$x^2 + 4x - 21 + 7 = 0$$

$$x^2 + 4x - 14 = 0 \quad \text{(standard form)}$$

**Question # 1 (iii)** $\dfrac{x}{x+1} + \dfrac{x+1}{x} = 6$

Sol//

$$\left(\frac{x}{x+1}\right) + \left(\frac{x+1}{x}\right) = 6$$

$$(x)\cancel{(x+1)}\left(\frac{x}{\cancel{x+1}}\right) + \cancel{(x)}(x+1)\left(\frac{x+1}{\cancel{x}}\right) = 6(x)(x+1)$$

$$(x)(x) + (x+1)(x+1) = 6(x)(x+1)$$

$$x^2 + x^2 + x + x + 1 = 6(x^2 + x)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$2x^2 - 6x^2 + 2x - 6x + 1 = 0$$

$$-4x^2 - 4x + 1 = 0$$

Divided by "-1"

$$\frac{-4x^2}{-1} \quad \frac{-4x}{-1} + \frac{1}{-1} = \frac{0}{-1}$$

$$4x^2 + 4x - 1 = 0$$ (standard form)

## Question # 1 (iv)



$$\frac{x+4}{x-2} - \frac{x-2}{x} + 4 = 0$$

Sol//

$$\left(\frac{x+4}{x-2}\right) - \left(\frac{x-2}{x}\right) + 4 = 0$$

$$(x)\cancel{(x-2)}\left(\frac{x+4}{\cancel{x-2}}\right) - \cancel{(x)}(x-2)\left(\frac{x-2}{\cancel{x}}\right) + 4(x)(x-2) = 0$$

$$(x)(x+4) - (x-2)(x-2) + 4x(x-2) = 0$$

$$x^2 + 4x - (x^2 - 2x - 2x + 4) + 4x^2 - 8x = 0$$

$$\cancel{x^2} + 4x - \cancel{x^2} + 2x + 2x - 4 + 4x^2 - 8x = 0$$

$4x^2 + \underline{4x + 2x + 2x} - 8x - 4 = 0$

$4x^2 + \cancel{8x} - \cancel{8x} - 4 = 0$

$4x^2 - 4 = 0$

Divided by "4"

$\frac{4x^2}{4} - \frac{4}{4} = \frac{0}{4}$

$x^2 - 1 = 0$ (Pure Quadratic Equation)

Q#2:- Solve by factorization:

Question #2 (ii)

$3y^2 = y(y-5)$



Sol//

$3y^2 = y(y-5)$

$3y^2 = y^2 - 5y$

$3y^2 - y^2 + 5y = 0$

$2y^2 + 5y = 0$

$y(2y+5) = 0$

$\boxed{y = 0}$

$2y + 5 = 0$

$2y = -5$

$\boxed{y = -\frac{5}{2}}$

$S.S = \{0, -\frac{5}{2}\}$ Answer

# Question #2 (iv)

$$x^2 - 11x = 152$$

**Sol//**

$$x^2 - 11x = 152$$

$$x^2 - \underline{11x} - 152 = 0$$

$$\underbrace{x^2 + 8x}_{\text{common}} \underbrace{-19x - 152}_{\text{common}} = 0$$

$-152x^2$: $\times 1$ 152, $\times 2$ 76, $\times 4$ 38; $\boxed{+8x \quad -19x}$ ✓

$$x(\underline{x+8}) - 19(\underline{x+8}) = 0$$

$$(x+8)(x-19) = 0$$

$x + 8 = 0$ | $x - 19 = 0$

$x = -8$ | $x = 19$



$S.S = \{-8, 19\}$ Answer.

# Question #2 (v)

$$\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$$

**Sol//**

$$\left(\frac{x+1}{x}\right) + \left(\frac{x}{x+1}\right) = \left(\frac{25}{12}\right)$$

$$(\cancel{x})(x+1)\left(\frac{x+1}{\cancel{x}}\right) + (x)(\cancel{x+1})\left(\frac{x}{\cancel{x+1}}\right) = \left(\frac{25}{12}\right)(x)(x+1)$$

$$(x+1)(x+1) + (x)(x) = \left(\frac{25}{12}\right)(x)(x+1)$$

$$x^2 + \underline{x + x} + 1 + x^2 = \frac{25}{12}(x^2 + x)$$

$$2x^2 + 2x + 1 = \frac{25}{12}(x^2 + x)$$

$$12(2x^2+2x+1) = 25(x^2+x)$$

$$24x^2 + 24x + 12 = 25x^2 + 25x$$

$$24x^2 - 25x^2 + 24x - 25x + 12 = 0$$

$$-1x^2 - 1x + 12 = 0$$

Divided by "$-1$"

$$\frac{-1x^2}{-1} - \frac{1x}{-1} + \frac{12}{-1} = \frac{0}{-1}$$

$$x^2 + x - 12 = 0$$

$-\boxed{12}\,x^2$

x 1 12

x 2 6

$\boxed{-3x \quad +4x}$

$$\underbrace{x^2 - 3x}_{\text{Common}} + \underbrace{4x - 12}_{\text{Common}} = 0$$



$$x(x-3) + 4(x-3) = 0$$

$$(x-3)(x+4) = 0$$

| | |
|---|---|
| $x-3 = 0$ | $x+4 = 0$ |
| $x = +3$ | $x = -4$ |

$S.S = \{-4, 3\}$ Answer

**Question # 3:- Solve the following equations by completing square:**

Question # 3 (i):-

$$7x^2 + 2x - 1 = 0$$

Sol//

$$7x^2 + 2x - 1 = 0$$

First of all we remove the coefficient of $x^2$, so we divided by 7

$$\frac{7x^2}{7} + \frac{2x}{7} - \frac{1}{7} = \frac{0}{7}$$

$$x^2 + \frac{2}{7}x - \frac{1}{7} = 0 \longrightarrow ①$$



$$\left[\frac{1}{2}(\ )\right]^2 \quad \text{formula}$$

$$= \left[\frac{1}{2}\left(\frac{2}{7}\right)\right]^2$$

$$= \left[\frac{1}{7}\right]^2$$

Add and Subtract $\left(\frac{1}{7}\right)^2$ in eq ①

$$x^2 + \frac{2}{7}x + \left(\frac{1}{7}\right)^2 - \left(\frac{1}{7}\right)^2 - \frac{1}{7} = 0$$

$$\left(x + \frac{1}{7}\right)^2 = \left(\frac{1}{7}\right)^2 + \frac{1}{7}$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{1}{49}+\frac{1}{7}$$

L.C.M

$$\left(x+\frac{1}{7}\right)^2 = \frac{1+7}{49}$$

$$\left(x+\frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking Square Root on both side.

$$\sqrt{\left(x+\frac{1}{7}\right)^2} = \sqrt{\frac{8}{49}}$$

$$x+\frac{1}{7} = \pm\frac{\sqrt{8}}{\sqrt{49}}$$



$$\sqrt{8} = \sqrt{4\times 2}$$
$$= \sqrt{4}\times\sqrt{2}$$
$$= 2\sqrt{2}$$

$$x+\frac{1}{7} = \pm\frac{2\sqrt{2}}{7}$$

$$\sqrt{49} = 7$$

$$x+\frac{1}{7} = +\frac{2\sqrt{2}}{7}$$

$$x = \frac{2\sqrt{2}}{7}-\frac{1}{7}$$

$$x = \frac{2\sqrt{2}-1}{7}$$

$$x+\frac{1}{7} = -\frac{2\sqrt{2}}{7}$$

$$x = -\frac{2\sqrt{2}}{7}-\frac{1}{7}$$

$$x = \frac{-2\sqrt{2}-1}{7}$$

$$S.S = \left\{\frac{2\sqrt{2}-1}{7}, \frac{-2\sqrt{2}-1}{7}\right\}$$

OR

$$= \left\{\frac{-1\pm 2\sqrt{2}}{7}\right\}$$

Answer Ans.

Question #3(v)

$$3x^2+7x=0$$

Sol//

$$3x^2+7x=0$$

First of all we remove the Coefficient of $x^2$, So we divided by 3.

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\left(\frac{7}{3}\right)x=0 \longrightarrow (1)$$

Formula

$$\left[\frac{1}{2}(\;)\right]^2$$



$$=\left[\frac{1}{2}\left(\frac{7}{3}\right)\right]^2$$

$$=\left[\frac{7}{6}\right]^2$$

Add & Subtract $\left(\frac{7}{6}\right)^2$ in eq (1)

$$x^2+\frac{7}{3}x+\left(\frac{7}{6}\right)^2-\left(\frac{7}{6}\right)^2=0$$

$$\left(x+\frac{7}{6}\right)^2-\left(\frac{7}{6}\right)^2=0$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking Square Root on both sides.

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\sqrt{\left(\frac{7}{6}\right)^2}$$

$$x+\frac{7}{6}=\pm\frac{7}{6}$$

$x+\frac{7}{6}=\frac{7}{6}$

$x=\frac{7}{6}-\frac{7}{6}$

$x=\frac{7-7}{6}$

$x=\frac{0}{6}$

$x=0$

$x+\frac{7}{6}=-\frac{7}{6}$

$x=-\frac{7}{6}-\frac{7}{6}$

$x=\frac{-7-7}{6}$

$x=-\frac{14}{6}$

Divided by 2

$x=-\frac{7}{3}$



$S.S=\left\{-\frac{7}{3},0\right\}$

## Question #3 (ix)

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$

**Sol,**

$$(4)-\left(\frac{8}{3x+1}\right)=\left(\frac{3x^2+5}{3x+1}\right)$$

$$(4)(3x+1)-\cancel{(3x+1)}\left(\frac{8}{\cancel{3x+1}}\right)=\cancel{(3x+1)}\left(\frac{3x^2+5}{\cancel{3x+1}}\right)$$

$$12x+4-8=3x^2+5$$

$$-3x^2+12x+4-8-5=0$$

$$-3x^2+12x-9=0$$

$-3x^2 + 12x - 9 = 0$

First of all, we remove the Coefficient of $x^2$, So we divided by "$-3$"

$$\frac{-3x^2}{-3} + \frac{12x}{-3} - \frac{9}{-3} = \frac{0}{-3}$$

$$x^2 - 4x + 3 = 0 \longrightarrow (1)$$

Formula

$$\left[\frac{1}{2}(\ )\right]^2$$

$$= \left[\frac{1}{2}(4)\right]^2$$

$$= [2]^2$$



Add and Subtract $[2]^2$ in eq (1)

$$x^2 - 4x + (2)^2 - (2)^2 + 3 = 0$$

$$(x-2)^2 - 4 + 3 = 0$$

$$(x-2)^2 = 4 - 3$$

$$(x-2)^2 = 1$$

Taking Square Root on both sides.

$$\sqrt{(x-2)^2} = \sqrt{1}$$

$$x - 2 = \pm 1$$

| | | |
|---|---|---|
| $x-2=1$ | $x-2=-1$ | $S.S=\{1,3\}$ |
| $x=1+2$ | $x=-1+2$ | Answer |
| $\boxed{x=3}$ | $\boxed{x=1}$ | |

# EXERCISE # 1.2

## Q#1 ( i , iii, vii , viii)

Solve the following equations using quadratic formula.

### Question # 1 (i)

$$2 - x^2 = 7x$$

Sol//

$$2 - x^2 = 7x$$

$$-x^2 - 7x + 2 = 0$$

Divided by "-1"

$$1x^2 + 7x - 2 = 0$$

Compare with standard form



$$ax^2 + bx + c = 0$$

$$a = 1 \quad , \quad b = 7 \quad , \quad c = -2$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2}$$

$$S.S = \left\{ \frac{-7 \pm \sqrt{57}}{2} \right\}$$ Answer.

## Question # 1 (iii)

$$\sqrt{3}x^2 + x = 4\sqrt{3}$$

Sol//

$$\sqrt{3}x^2 + x = 4\sqrt{3}$$

$$\sqrt{3}x^2 + 1x - 4\sqrt{3} = 0$$

Compare with standard form.

$$ax^2 + bx + c = 0$$

$$a = \sqrt{3}, \quad b = 1, \quad c = -4\sqrt{3}$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(\sqrt{3})^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1+48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$x = \frac{-1+7}{2\sqrt{3}}$$

$$x = \frac{6}{2\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}}$$

$$x = \frac{(\sqrt{3})^2}{\sqrt{3}}$$

$$x = \frac{(\sqrt{3})\cancel{(\sqrt{3})}}{\cancel{\sqrt{3}}}$$

$$x = \sqrt{3}$$

$$x = \frac{-1-7}{2\sqrt{3}}$$

$$x = \frac{-8}{2\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}}$$

$$S.S = \left\{-\frac{4}{\sqrt{3}}, \sqrt{3}\right\}$$ Answer.

## Question #1 (vii)



$$\frac{3}{x-6} - \frac{4}{x-5} = 1$$

Sol/

$$\left(\frac{3}{x-6}\right) - \left(\frac{4}{x-5}\right) = 1$$

$$(\cancel{x-6})(x-5)\left(\frac{3}{\cancel{x-6}}\right) - (x-6)(\cancel{x-5})\left(\frac{4}{\cancel{x-5}}\right) = 1(x-6)(x-5)$$

$$(x-5)(3) - (x-6)(4) = 1(x-6)(x-5)$$

$$3x - 15 - (4x-24) = x^2 - 5x - 6x + 30$$

$$3x - 15 - 4x + 24 = x^2 - 5x - 6x + 30$$

$$-x^2 + \underline{3x - 4x + 5x + 6x} \; \underline{-15 + 24 - 30} = 0$$

$$-x^2 + 10x - 21 = 0$$

Divided by "$-1$"

$1x^2 - 10x + 21 = 0$

Compare with standard form

$ax^2 + bx + c = 0$

$a = 1$ , $b = -10$ , $c = 21$

**Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{+10 \pm \sqrt{100 - 84}}{2}$$



$$x = \frac{10 \pm \sqrt{16}}{2}$$

$\because \sqrt{16} = \sqrt{4^2} = 4$

$$x = \frac{10 \pm 4}{2}$$

$x = \frac{10+4}{2}$ | $x = \frac{10-4}{2}$

$x = \frac{14}{2}$ | $x = \frac{6}{2}$

$x = 7$ | $x = 3$

$$S.S = \{3, 7\}$$

Answer

## Question # 1 (viii)

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$$

Sol//

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$$

Multiply by $3(x-1)(2x)$

$$3\cancel{(x-1)}(2x)\left(\frac{x+2}{\cancel{x-1}}\right) - 3(x-1)\cancel{(2x)}\left(\frac{4-x}{\cancel{2x}}\right) = \frac{7}{\cancel{3}}\cancel{3}(x-1)(2x)$$

$$3(2x)(x+2) - 3(x-1)(4-x) = 7(x-1)(2x)$$

$$3(2x^2+4x) - 3(4x-x^2-4+x) = 7(2x^2-2x)$$



$$6x^2+\cancel{12x}-\cancel{12x}+3x^2+12-3x = 14x^2-14x$$

$$6x^2+3x^2-14x^2-3x+14x+12 = 0$$

$$-5x^2+11x+12 = 0$$

Divided by "$-1$"

$$\frac{-5x^2}{-1}+\frac{11x}{-1}+\frac{12}{-1} = \frac{0}{-1}$$

$$5x^2-11x-12 = 0$$

Compare with Quadratic Standard form

$$ax^2+bx+c = 0$$

$$a = 5, \quad b = -11, \quad c = -12$$

Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$$

Put $a = 5$ , $b = -11$ , $c = -12$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(5)(-12)}}{2(5)}$$

$$x = \frac{+11 \pm \sqrt{121 + 240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10}$$

$$x = \frac{11 \pm 19}{10}$$ $\because \sqrt{361} = \sqrt{(19)^2} = 19$



$$x = \frac{11 + 19}{10}$$

$$x = \frac{30}{10}$$

$$x = 3$$

$$x = \frac{11 - 19}{10}$$

$$x = \frac{-8}{10}$$ Divided by "2"

$$x = -\frac{4}{5}$$

$$S.S = \left\{ -\frac{4}{5}, 3 \right\}$$

Answer

# EXERCISE # 1.3

## Question # 2, 7, 9, 10, 12, 14

Solve the following equation

## Question # 2:-

$$2x^4 = 9x^2 - 4$$

Sol//

$$2x^4 = 9x^2 - 4$$

$$2x^4 - 9x^2 + 4 = 0 \longrightarrow (1)$$

Let $y = x^2$

Squaring on both sides



$$(y)^2 = (x^2)^2$$

$$y^2 = x^4$$

put the value of $x^2$ & $x^4$ in eq (1)

$$2y^2 - 9y + 4 = 0$$

Now we solve by factorization Method

$$2y^2 - 1y - 8y + 4 = 0$$

$(2y^2)(4)$

$= 8y^2$

$-1y \quad -8y$ ✓

$2 \quad 4$ ✗

$$y(2y-1) - 4(2y-1) = 0$$

$$(2y-1)(y-4) = 0$$

$2y - 1 = 0$ | $y - 4 = 0$

$2y = 1$ | $y = 4$

$y = \frac{1}{2}$

Now put the value of $y = x^2$

$x^2 = \frac{1}{2}$ | $x^2 = 4$

Taking square root

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$ | $\sqrt{x^2} = \sqrt{4}$

$x = \pm \frac{1}{\sqrt{2}}$ | $x = \pm 2$

$$S.S = \left\{ \pm \frac{1}{\sqrt{2}}, \pm 2 \right\}$$ Answer. Sir.



## Question # 7:-

$$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$$

Sol//

$$\left(\frac{x}{x-3}\right) + 4\left(\frac{x-3}{x}\right) = 4 \longrightarrow (1)$$

Let $y = \frac{x}{x-3}$

then $\frac{1}{y} = \frac{x-3}{x}$

Now put the value of $\left(\frac{x}{x-3}\right)$ and $\left(\frac{x-3}{x}\right)$ in equation 1.

$$y + 4\left(\frac{1}{y}\right) = 4$$

$$y + \frac{4}{y} = 4$$

Multiply by "y" on both sides.

$$y(y) + \cancel{y}\left(\frac{4}{\cancel{y}}\right) = 4y$$

$$y^2 + 4 = 4y$$

$$1y^2 - 4y + 4 = 0$$

Compare with standard form

$$ay^2 + by + c = 0$$



$a = 1$ , $b = -4$ , $c = 4$

Quadratic formula in "y"

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$$

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$y = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$y = \frac{4 \pm \sqrt{0}}{2}$$

$$y = \frac{4}{2}$$

$$y = 2$$

Now put the value of $y = \frac{x}{x-3}$

$$\frac{x}{x-3} = 2$$

$$x = 2(x-3)$$

$$x = 2x - 6$$

$$x - 2x = -6$$



$$-1x = -6$$

$$x = \frac{-6}{-1}$$

$$x = 6$$

$$S.S = \{6\}$$ Answer

## Question # 9:-

$$\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$$

Sol/

$$\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12} \longrightarrow (1)$$

let

$$y = \frac{x-a}{x+a}$$

then $\frac{1}{y} = \frac{x+a}{x-a}$

put the value of $\frac{x-a}{x+a}$ & $\frac{x+a}{x-a}$ in eq (1)

$$y - \frac{1}{y} = \frac{7}{12}$$



Multiply by 12y on both side

$$12y(y) - 12y\left(\frac{1}{y}\right) = \left(\frac{7}{\cancel{12}}\right)\cancel{12}y$$

$$12y^2 - 12 = 7y$$

$$12y^2 - 7y - 12 = 0$$

$(12y^2)(-12)$

$-144y^2$

| | |
|---|---|
| 1 | 144 X |
| 2 | 72 X |
| 4 | 36 X |
| 6 | 24 X |
| 8 | 18 X |
| +9y | −16y ✓ |

$$12y^2 + 9y - 16y - 12 = 0$$

$$3y(4y+3) - 4(4y+3) = 0$$

$$(4y+3)(3y-4) = 0$$

$$4y+3=0$$
$$4y=-3$$
$$y=-\frac{3}{4}$$

$$3y-4=0$$
$$3y=4$$
$$y=\frac{4}{3}$$

Now Put the value of $y=\frac{x-a}{x+a}$

$$\frac{x-a}{x+a}=-\frac{3}{4}$$
$$4(x-a)=-3(x+a)$$
$$4x-4a=-3x-3a$$
$$4x+3x=-3a+4a$$
$$7x=1a$$
$$x=\frac{a}{7}$$

$$\frac{x-a}{x+a}=\frac{4}{3}$$
$$3(x-a)=4(x+a)$$
$$3x-3a=4x+4a$$
$$3x-4x=4a+3a$$
$$-x=7a$$
$$x=-7a$$



$$S.S=\left\{-7a,\frac{a}{7}\right\}$$ Answer

## Question #10:-

$$x^4-2x^3-2x^2+2x+1=0$$

Sol//

$$x^4-2x^3-2x^2+2x+1=0$$

Divided by "$x^2$" on both side

$$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$

✓ ✓

$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$

Rearrange

$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$

$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{1}{x}\right) - 2 = 0 \longrightarrow ①$

Let $\boxed{y = x - \frac{1}{x}}$

Squaring Both side

$(y)^2 = \left(x - \frac{1}{x}\right)^2$

using formula
$(a-b)^2 = a^2 + b^2 - 2ab$

$y^2 = (x)^2 + \left(\frac{1}{x}\right)^2 - 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)$



$y^2 = x^2 + \frac{1}{x^2} - 2$

$\boxed{y^2 + 2 = x^2 + \frac{1}{x^2}}$

Now put the value of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$ in eq ①

$(y^2 + 2) - 2(y) - 2 = 0$

$y^2 + \cancel{2} - 2y - \cancel{2} = 0$

$y^2 - 2y = 0$

$y(y-2) = 0$

$y = 0$ | $y - 2 = 0$

$y = 2$

Now put the value of $y = x - \frac{1}{x}$

$x - \frac{1}{x} = 0$

Multiply by $x$

$x(x) - x\left(\frac{1}{x}\right) = (0)x$

$x^2 - 1 = 0$

$x^2 = 1$

Taking square root

$\sqrt{x^2} = \sqrt{1}$

$\boxed{x = \pm 1}$

$x - \frac{1}{x} = 2$

Multiply by $x$

$x(x) - \cancel{x}\left(\frac{1}{\cancel{x}}\right) = (2)x$

$x^2 - 1 = 2x$

$1x^2 - 2x - 1 = 0$

Compare with standard form

$ax^2 + bx + c = 0$



$a = 1$, $b = -2$, $c = -1$

Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4 + 4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

Now

$x = \frac{2+2\sqrt{2}}{2}$ OR $x = \frac{2-2\sqrt{2}}{2}$

$x = \frac{\cancel{2}(1+\sqrt{2})}{\cancel{2}}$ , $x = \frac{\cancel{2}(1-\sqrt{2})}{\cancel{2}}$

$\boxed{x = 1+\sqrt{2}}$ , $\boxed{x = 1-\sqrt{2}}$

$S.S = \{\pm 1, 1+\sqrt{2}, 1-\sqrt{2}\}$

OR

$S.S = \{\pm 1, 1\pm\sqrt{2}\}$ Answer.

## Question #12 :-

$$4.2^{2x+1} - 9.2^{x} + 1 = 0$$



Sol//

$$4.2^{2x+1} - 9.2^{x} + 1 = 0$$

$$4.2^{2x}.2^{1} - 9.2^{x} + 1 = 0$$

Multiply

$$8.2^{2x} - 9.2^{x} + 1 = 0 \longrightarrow (1)$$

Let

$$y = 2^{x}$$

Taking square on both sides

$$(y)^2 = (2^x)^2$$

$$y^2 = 2^{2x}$$

Now put the value of $2^x$ and $2^{2x}$ in eq (1)

$$8y^2 - 9y + 1 = 0$$

Multiply

$$8y^2 - 9y + 1 = 0$$

$(8y^2)(1)$

$= 8y^2$

$-1y \quad -8y$

$$8y^2 - 1y - 8y + 1 = 0$$

$$y(8y-1) - 1(8y-1) = 0$$

$$(8y-1)(y-1) = 0$$

| | |
|---|---|
| $8y - 1 = 0$ | $y - 1 = 0$ |
| $8y = 1$ | $y = 1$ |
| $y = \frac{1}{8}$ | |



Now put the value of $y = 2^x$

| | |
|---|---|
| $2^x = \frac{1}{8}$ | $2^x = 1$ |
| $2^x = \frac{1}{2^3}$ | put $1 = 2^0$ |
| $\cancel{2}^x = \cancel{2}^{-3}$ | $\cancel{2}^x = \cancel{2}^0$ |
| $x = -3$ | $x = 0$ |

$$S.S = \{-3, 0\}$$

Answer

<u>**Question # 14:-**</u>

$$2^x + 64.2^{-x} - 20 = 0$$

Sol/

$$2^x + 64.2^{-x} - 20 = 0$$

$$2^x + 64.\frac{1}{2^x} - 20 = 0 \longrightarrow (1)$$

let $y = 2^x$

(1) $\Rightarrow$

$$y + 64.\frac{1}{y} - 20 = 0$$

Multiply by "$y$" on both sides



$$y(y) + y(64.\frac{1}{y}) - y(20) = (0)y$$

$$y^2 + 64 - 20y = 0$$

Rearrange

$$y^2 - 20y + 64 = 0$$

Multiply

$$(y^2)(64)$$

$$= 64y^2$$

| | | |
|---|---|---|
| 1 | 64 | X |
| 2 | 32 | X |
| $-4y$ | $-16y$ | ✓ |

$$y^2 - 4y - 16y + 64 = 0$$

$$y(y-4) - 16(y-4) = 0$$

$$(y-4)(y-16) = 0$$

$y - 4 = 0$ | $y - 16 = 0$

$y = 4$ | $y = 16$

Now put the value of $y = 2^x$

$2^x = 4$ | $2^x = 16$

$\cancel{2}^x = \cancel{2}^2 \quad \because 4 = 2^2$ | $\cancel{2}^x = \cancel{2}^4 \quad \because 16 = 2^4$

$\boxed{x = 2}$ | $\boxed{x = 4}$

$S.S = \{2, 4\}$

# EXERCISE # 1.4

## Question (1, 3, 8, 9)

Solve the following equation

## Question # 1:-

$$2x+5=\sqrt{7x+16}$$

Sol//

$$2x+5=\sqrt{7x+16}$$

Taking Square on both sides

$$(2x+5)^2=(\sqrt{7x+16})^2$$

$$(2x)^2+(5)^2+2(2x)(5)=7x+16$$

$$4x^2+25+20x=7x+16$$



$$4x^2+20x-7x+25-16=0$$

$$4x^2+13x+9=0$$

$$4x^2+4x+9x+9=0$$

$$4x(x+1)+9(x+1)=0$$

$$(x+1)(4x+9)=0$$

$(4x^2)(9)$

$=36x^2$

| | |
|---|---|
| 1 | 36 ✗ |
| 2 | 18 ✗ |
| 3 | 12 ✗ |
| $+4x$ | $+9x$ ✓ |

$x+1=0$

$x=-1$

$4x+9=0$

$4x=-9$

$x=\frac{-9}{4}$

$$S.S=\left\{-1,-\frac{9}{4}\right\}$$ Answer

## Question # 3:-

$$4x = \sqrt{13x+14} - 3$$

Sol

$$4x = \sqrt{13x+14} - 3$$

$$4x + 3 = \sqrt{13x+14}$$

Taking Square on both side

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$

$$(4x)^2 + (3)^2 + 2(4x)(3) = 13x + 14$$

$$16x^2 + 9 + 24x = 13x + 14$$

$$16x^2 + 24x - 13x + 9 - 14 = 0$$



$$16x^2 + 11x - 5 = 0$$

$(16x^2)(-5)$

$-80x^2$

| | |
|---|---|
| 1 | 80 ✗ |
| 2 | 40 ✗ |
| 4 | 20 ✗ |
| $-5x$ | $+16x$ ✓ |

$$16x^2 - 5x + 16x - 5 = 0$$

$$x(16x-5) + 1(16x-5) = 0$$

$$(16x-5)(x+1) = 0$$

$16x - 5 = 0$ | $x + 1 = 0$

$16x = 5$ | $x = -1$

$x = \frac{5}{16}$

$$S.S = \left\{-1, \frac{5}{16}\right\}$$ Answer.

## Question # 8 :-

$$\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$$

Sol/

$$\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$$

Taking Square on both sides

$$\left(\sqrt{4a+x} - \sqrt{a-x}\right)^2 = \left(\sqrt{a}\right)^2$$

By using formula

$$(a-b)^2 = (a)^2 + (b)^2 - 2(a)(b)$$

$$\left(\sqrt{4a+x}\right)^2 + \left(\sqrt{a-x}\right)^2 - 2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right) = a$$

$$4a+\cancel{x} + a - \cancel{x} - 2\left(\sqrt{4a+x}\sqrt{a-x}\right) = a$$



$$5a - 2\sqrt{(4a+x)(a-x)} = a$$

$$-2\sqrt{(4a+x)(a-x)} = a - 5a$$

$$-2\sqrt{(4a+x)(a-x)} = -4a$$

$$\sqrt{(4a+x)(a-x)} = \frac{-4a}{-2}$$

$$\sqrt{(4a+x)(a-x)} = 2a$$

Again Taking square root on both sides

$$\left(\sqrt{(4a+x)(a-x)}\right)^2 = (2a)^2$$

$$(4a+x)(a-x) = 4a^2$$

$$4a^2 - 4ax + ax - x^2 = 4a^2$$

$$\cancel{4a^2} - 4ax + 1ax - x^2 - \cancel{4a^2} = 0$$

$$-4ax + 1ax - x^2 = 0$$

$$-3ax - x^2 = 0$$

$$x(-3a - x) = 0$$

$x = 0$ | $-3a - x = 0$

$$-x = 3a$$

$$x = -3a$$

$$S.S = \{-3a, 0\}$$ Answer

## Question #9:-

$$\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$$



Sol//

$$\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$$

Let $y = x^2 + x$

$$\sqrt{y+1} - \sqrt{y-1} = 1$$

$$\sqrt{y+1} = 1 + \sqrt{y-1}$$

Taking Square on both side

$$(\sqrt{y+1})^2 = (1 + \sqrt{y-1})^2$$

Applying formula $(a+b)^2 = (a)^2 + (b)^2 + 2(a)(b)$

$$y+1 = (1)^2 + (\sqrt{y-1})^2 + 2(1)(\sqrt{y-1})$$

$$y + \cancel{1} = \cancel{1} + y - \cancel{1} + 2\sqrt{y-1}$$

$$y + 1 - \cancel{y} = 2\sqrt{y-1}$$

$1 = 2\sqrt{y-1}$

Again Taking Square on both side

$(1)^2 = (2\sqrt{y-1})^2$

$1 = (2)^2(\sqrt{y-1})^2$

$1 = 4(y-1)$

put the value of $y = x^2 + x$

$1 = 4(x^2 + x - 1)$

$1 = 4x^2 + 4x - 4$



$0 = 4x^2 + 4x - 4 - 1$

$0 = 4x^2 + 4x - 5$

$4x^2 + 4x - 5 = 0$

$ax^2 + bx + c = 0$

$a = 4$ , $b = 4$ , $c = -5$

Using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) + \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16+80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$96 = 16 \times 6$

$$x = \frac{-4 \pm \sqrt{16 \times 6}}{8}$$

$$x = \frac{-4 \pm \sqrt{16}\sqrt{6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$\sqrt{16} = \sqrt{4^2} = 4$

$$x = \frac{\cancel{4}(-1 \pm \sqrt{6})}{\cancel{8}_{2}}$$



$$x = \frac{-1 \pm \sqrt{6}}{2}$$

$$S.S = \left\{ \frac{-1 \pm \sqrt{6}}{2} \right\}$$ Answer sir.

# Unit #1

## Miscellaneous EXERCISE – 1

### Q#1:- Multiple Choice Questions

(i) Standard form of quadratic equation is

$$ax^2 + bx + c = 0$$

(ii) The number of terms in a standard quadratic equation $ax^2 + bx + c = 0$ is

3

(iii) The number of methods to solve a quadratic equation is



3

(iv) The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(v) Two linear factors of $x^2 - 15x + 56$ are

$(x-7)$ and $(x-8)$

(vi) An equation, which remains unchanged when $x$ is replaced by $\frac{1}{x}$ is called a/an

Reciprocal Equation

(vii) An equation of the type

$3^{x} + 3^{2-x} + 6 = 0$ is a/an

Exponential Equation

(viii) The solution set of equation $4x^2 - 16 = 0$ is

$\{\pm 2\}$

(ix) An equation of the form

$2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$

is called a/an

Reciprocal Equation



## Question #2 :-

Write short answers of the following questions.

Q#2(i):- Solve $x^2 + 2x - 2 = 0$

Sol//

$x^2 + 2x - 2 = 0$

Compare with standard form

$ax^2 + bx + c = 0$

$a = 1 \quad , \quad b = 2 \quad , \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+8}}{2}$$

$$x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \times 3}}{2}$$ $12 = 4 \times 3$

$$x = \frac{-2 \pm \sqrt{4}\sqrt{3}}{2}$$ $\sqrt{4} = \sqrt{2^2} = 2$

$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$x = \frac{\cancel{2}(-1 \pm \sqrt{3})}{\cancel{2}}$$



$$x = -1 \pm \sqrt{3}$$

$$S.S = \{-1 \pm \sqrt{3}\}$$ Answer

Q#2(ii) Solve by factorization

$$5x^2 = 15x$$

Sol//

$$5x^2 = 15x$$

$$5x^2 - 15x = 0$$

$$5x(x-3) = 0$$

| | |
|---|---|
| $5x = 0$ | $x - 3 = 0$ |
| $x = \frac{0}{5}$ | $x = 3$ |
| $x = 0$ | |

$$S.S = \{0, 3\}$$ Answer

Q#2 (iii) Write in standard form

$$\frac{1}{x+4} + \frac{1}{x-4} = 3$$

Sol//

$$\frac{1}{x+4} + \frac{1}{x-4} = 3$$

$$(\cancel{x+4})(x-4)\left(\frac{1}{\cancel{x+4}}\right) + (x+4)(\cancel{x-4})\left(\frac{1}{\cancel{x-4}}\right) = 3(x+4)(x-4)$$

$$(x-4)(1) + (x+4)(1) = 3(x+4)(x-4)$$

$$x - \cancel{4} + x + \cancel{4} = 3(x^2 - \cancel{4x} + \cancel{4x} - 16)$$

$$2x = 3(x^2 - 16)$$

$$2x = 3x^2 - 48$$



$$-3x^2 + 2x + 48 = 0$$

Multiply by "-1"

$$-1(-3x^2 + 2x + 48) = -1(0)$$

$$3x^2 - 2x - 48 = 0 \quad \text{standard form}$$

$$ax^2 + bx + c = 0$$

Q#2 (iv) Write the names of the methods for solving a quadratic equation.

Ans:-

(i) Factorization

(ii) Completing square

(iii) Quadratic Formula.

Q#2(v) Solve $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Sol//

$$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$$

Taking square root on both sides

$$\sqrt{\left(2x-\frac{1}{2}\right)^2}=\sqrt{\frac{9}{4}}$$

$$2x-\frac{1}{2}=\pm\frac{3}{2} \qquad \because \sqrt{9}=3, \quad \sqrt{4}=2$$



| | |
|---|---|
| $2x-\frac{1}{2}=\frac{3}{2}$ | $2x-\frac{1}{2}=-\frac{3}{2}$ |
| $2x=\frac{3}{2}+\frac{1}{2}$ | $2x=-\frac{3}{2}+\frac{1}{2}$ |
| $2x=\frac{3+1}{2}$ | $2x=\frac{-3+1}{2}$ |
| $2x=\frac{4}{2}$ | $2x=-\frac{2}{2}$ |
| $2x=2$ | $2x=-1$ |
| $x=\frac{2}{2}$ | $x=-\frac{1}{2}$ |
| $x=1$ | |

$$S.S=\left\{-\frac{1}{2},1\right\}$$ Answer

Q#2 (vi) Solve $\sqrt{3x+18} = x$

Sol//

$$\sqrt{3x+18} = x$$

Taking square on both sides.

$$\left(\sqrt{3x+18}\right)^2 = (x)^2$$

$$3x + 18 = x^2$$

$$-x^2 + 3x + 18 = 0$$

Multiply by "-1"

$$-1\left(-x^2 + 3x + 18\right) = -1(0)$$



$$x^2 - 3x - 18 = 0$$

$$x^2 + 3x - 6x - 18 = 0$$

$$x(x+3) - 6(x+3) = 0$$

$$(x+3)(x-6) = 0$$

$(x^2)(-18)$

$-18x^2$

1 18 ✗

2 9 ✗

$+3x$ $-6x$ ✓

| $x+3=0$ | $x-6=0$ |
|---|---|
| $x = -3$ | $x = 6$ |

$$S.S = \{-3, 6\}$$

Answer

$\neg$it.

## Q#2 (vii) Define Quadratic Equation?

**Ans:-** An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation or an equation of the second degree.

Example: $ax^2 + bx + c = 0$

$$2x^2 + 5x + 7 = 0$$

## Q#2 (viii) Define Reciprocal Equation?

**Ans:-** An equation is said to be reciprocal equation, if it remains unchanged, when $x$ is replaced by $\frac{1}{x}$.



Example:

$$x + \frac{1}{x} + 7 = 0$$

## Q#2 (ix) Define exponential Equation?

**Ans:-** An equation is said to be exponential equation, if variable occurs in exponent.

Example

$$5^x + 5^{-x} = 4$$

Q#2(x) Define radical equation?

Ans:- An equation involving expression under the radical sign is called a radical equation.

Example $\sqrt{x+3} = x+1$

Unit #2 Mathematics (10th)

# Theory of Quadratic Equations

## EXERCISE # 2.1

Q#1 (ii, iv) , Q#2 (i, iv)

Q #3 , Q#4 (iii) , Q#10

**Question #1:-** Find the discriminat of the following given quadratic equation.

(ii) $6x^2 - 8x + 3 = 0$



Sol// $6x^2 - 8x + 3 = 0$

Compare with Quadratic Standard form.

$ax^2 + bx + c = 0$

$a = 6$ , $b = -8$ , $c = 3$

Discriminant formula

$$\text{Disc.} = b^2 - 4ac$$

$$= (-8)^2 - 4(6)(3)$$

$$= 64 - 72$$

$$= \boxed{-8}$$ Answer

Q#1 (iv) $4x^2 - 7x - 2 = 0$

Sol//

$$4x^2 - 7x - 2 = 0$$

Compare with Quadratic Standand form.

$$ax^2 + bx + c = 0$$

$$a = 4 \quad , \quad b = -7 \quad , \quad c = -2$$

Discriminant formula

$$\text{Disc.} = b^2 - 4ac$$

$$= (-7)^2 - 4(4)(-2)$$

$$= 49 + 32$$



$$= \boxed{81}$$ Answer

Question#2:- Find the nature of the roots of the following given Quadratic equation and verify the result by Solving the equation

(i) $x^2 - 23x + 120 = 0$

Sol//

$$1x^2 - 23x + 120 = 0$$

Compare with Quadratic Standard form

$$ax^2 + bx + c = 0$$

$$a = 1 \ , \ b = -23 \ , \ c = 120$$

Discriminant formula

$$\text{Disc.} = b^2 - 4ac$$

$$= (-23)^2 - 4(1)(120)$$

$$= 529 - 480$$

$$= \boxed{49}$$

49 is a perfect square, so Roots are Rational (Real) and unequal.

**Verification by Solving the equation**

$$1x^2 - 23x + 120 = 0$$

$$ax^2 + bx + c = 0$$



$$\Rightarrow \quad a = 1 \quad , \quad b = -23 \quad , \quad c = 120$$

Using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-23) \pm \sqrt{(-23)^2 - 4(1)(120)}}{2(1)}$$

$$x = \frac{23 \pm \sqrt{529 - 480}}{2}$$

$$x = \frac{23 \pm \sqrt{49}}{2}$$

$$x = \frac{23 \pm 7}{2}$$

$x = \frac{23+7}{2}$ or $x = \frac{23-7}{2}$

$x = \frac{30}{2}$ | $x = \frac{16}{2}$

$\boxed{x = 15}$ | $\boxed{x = 8}$

So the roots are Rational (Real) and unequal.

## Question # 2 (iv)

$$3x^2 + 7x - 13 = 0$$

Sol//

$$3x^2 + 7x - 13 = 0$$
$$ax^2 + bx + c = 0$$

$a = 3,\ b = 7,\ c = -13$



Discriminant formula

$$\text{Disc.} = b^2 - 4ac$$
$$= (7)^2 - 4(3)(-13)$$
$$= 49 + 156$$
$$= \boxed{205}$$

205 is not a Perfect square So roots are Irrational (Real) and unequal.

## Verification:

$$3x^2 + 7x - 13 = 0$$
$$ax^2 + bx + c = 0$$

$a = 3$ , $b = 7$ , $c = -13$

Using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(-13)}}{2(3)}$$

$$x = \frac{-7 \pm \sqrt{49 + 156}}{6}$$



$$x = \frac{-7 \pm \sqrt{205}}{6}$$

So, roots are Irrational (Real) and unequal.

## Question # 3:-

For what value of $K$, the expression $K^2x^2 + 2(K+1)x + 4$ is perfect square

Sol//

$$K^2x^2 + 2(K+1)x + 4 = 0$$
$$ax^2 + bx + c = 0$$

$a = K^2$ , $b = 2(K+1)$ , $c = 4$

$a = K^2$ , $b = 2(K+1)$ , $c = 4$

Discriminant formula

$$\text{Disc.} = b^2 - 4ac$$

$$\text{Disc.} = [2(K+1)]^2 - 4(K^2)(4)$$

$$\text{Disc.} = (2)^2(K+1)^2 - 16K^2$$

$$\text{Disc.} = 4(K+1)^2 - 16K^2$$

applying formula $(a+b)^2 = (a)^2 + (b)^2 + 2(a)(b)$

$$\text{Disc.} = 4\left[(K)^2 + (1)^2 + 2(K)(1)\right] - 16K^2$$

$$\text{Disc.} = 4(K^2 + 1 + 2K) - 16K^2$$



$$\text{Disc.} = 4K^2 + 4 + 8K - 16K^2$$

$$\boxed{\text{Disc.} = -12K^2 + 8K + 4}$$

According to Given Condition

put

$$\text{Disc.} = 0$$

$$-12K^2 + 8K + 4 = 0$$

Multiply by "$-1$"

$$-1(-12K^2 + 8K + 4) = -1(0)$$

$$12K^2 - 8K - 4 = 0$$

Now Divided by "4" on both sides

$$\frac{12K^2}{4} - \frac{8K}{4} - \frac{4}{4} = \frac{0}{4}$$

$$3K^2 - 2K - 1 = 0$$

Solve by factorization

$(3K^2)(1)$

$3K^2$

$+1K \quad -3K$

$$3K^2 + 1K - 3K - 1 = 0$$

$$K(3K+1) - 1(3K+1) = 0$$

$$(3K+1)(K-1) = 0$$

$3K+1 = 0 \quad$ OR $\quad K-1 = 0$

$3K = -1 \quad \boxed{K = 1}$

$\boxed{K = \frac{-1}{3}}$

Answer Sir.

## Question # 4 :-

Find the value of K, if the roots of the following equations are equal.

(iii) $(3K+2)x^2 - 5(K+1)x + (2K+3) = 0$

Sol// $(3K+2)x^2 - 5(K+1)x + (2K+3) = 0$

$ax^2 + bx + c = 0$

$a = 3K+2, \quad b = -5(K+1), \quad c = 2K+3$

According to Given Condition

put

$$Disc. = 0$$

$$b^2 - 4ac = 0$$

put the value of a, b and c

$$[-5(K+1)]^2 - 4(3K+2)(2K+3) = 0$$

$$(-5)^2(K+1)^2 - 4(6K^2 + 9K + 4K + 6) = 0$$

applying formula

$$(a+b)^2 = (a)^2 + (b)^2 + 2(a)(b)$$

$$25\left[(K)^2 + (1)^2 + 2(K)(1)\right] - 4(6K^2 + 13K + 6) = 0$$



$$25(K^2 + 1 + 2K) - 4(6K^2 + 13K + 6) = 0$$

$$25K^2 + 25 + 50K - 24K^2 - 52K - 24 = 0$$

$$\underline{25K^2 - 24K^2} + \underline{50K - 52K} + \underline{25 - 24} = 0$$

$$1K^2 - 2K + 1 = 0$$

$$(K)^2 - 2(K)(1) + (1)^2 = 0$$

using

$$(a)^2 - 2(a)(b) + (b)^2 = (a-b)^2$$

$$(K-1)^2 = 0$$

OR

2nd Method

By factorization

$$K^2 - K - K + 1 = 0$$

$$K(K-1) - 1(K-1) = 0$$

$$(K-1)(K-1) = 0$$

$$(K-1)^2 = 0$$

Taking Square Root

$$\sqrt{(K-1)^2} = \sqrt{0}$$

$$K - 1 = 0$$

$$\boxed{K = 1}$$ Answer

**Question # 10:-**
Show that the roots of the equation

$(b-c)x^2+(c-a)x+(a-b)=0$ are real.

Sol// $(b-c)x^2+(c-a)x+(a-b)=0$

$a'x^2 + b'x + c' = 0$

$a'=(b-c)$ , $b'=(c-a)$ , $c'=(a-b)$

$\text{Disc.} = (b')^2 - 4(a')(c')$

$\text{Disc.} = (c-a)^2 - 4(b-c)(a-b)$

$\text{Disc.} = (c-a)^2 - 4(ab - b^2 - ca + bc)$



applying formula
$(a-b)^2 = (a)^2 + (b)^2 - 2(a)(b)$

$\text{Disc.} = \left[(c)^2+(a)^2-2(c)(a)\right] - 4ab + 4b^2 + 4ac - 4bc$

$\text{Disc.} = c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ac - 4bc$

$\text{Disc.} = c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc$

$\text{Disc.} = (c)^2 + (a)^2 + (-2b)^2 + 2(c)(a) + 2(a)(-2b) + 2(-2b)(c)$

Applying formula
$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$

$\text{Disc} = [c + a + (-2b)]^2$

$\boxed{\text{Disc} = (c+a-2b)^2}$ It is a perfect square, So roots are Real.

# EXERCISE # 2.2

## Question # 1 , 2(ii, viii), 3 , 4

**Q#1:-** Find the cube root of "$-1$".

**Sol//**

Let

$$x = (-1)^{1/3}$$

Taking cube on both side

$$[x]^3 = \left[(-1)^{1/3}\right]^3$$

$$x^3 = -1$$



$$x^3 + 1 = 0$$

Applying formula

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$(x+1)\left((x)^2 - (x)(1) + (1)^2\right) = 0$$

$$(x+1)(x^2 - x + 1) = 0$$

$x + 1 = 0$

$\boxed{x = -1}$

$x^2 - x + 1 = 0$

Compare with

$ax^2 + bx + c = 0$

$a = 1 \ , \ b = -1 \ , \ c = 1$

Applying Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm \sqrt{-1}\sqrt{3}}{2} \qquad \sqrt{-1} = i$$

$$x = \frac{1 \pm i\sqrt{3}}{2}$$

We know that



$$-\omega = \frac{1 + i\sqrt{3}}{2} \qquad -\omega^2 = \frac{1 - i\sqrt{3}}{2}$$

$$\boxed{x = -\omega} \quad \text{and} \quad \boxed{x = -\omega^2}$$

Q#1:- Find the cube root of "8".

Sol,

$$x = (8)^{1/3}$$

Taking cube on both sides

$$[x]^3 = \left[(8)^{1/3}\right]^3$$

$$x^3 = 8$$

Scanned with CamScanner

$$x^3 - 8 = 0$$

$$(x)^3 - (2)^3 = 0$$

$8 = 2\times2\times2$

$8 = (2)^3$

Applying formula

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-2)\left((x)^2 + (x)(2) + (2)^2\right) = 0$$

$$(x-2)(x^2 + 2x + 4) = 0$$

$x - 2 = 0$

$\boxed{x = 2}$

$x^2 + 2x + 4 = 0$

Compare with

$ax^2 + bx + c = 0$

$a = 1,\ b = 2,\ c = 4$



Apply Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm \sqrt{-4 \times 3}}{2}$$

$$x = \frac{-2 \pm \sqrt{-1}\sqrt{4}\sqrt{3}}{2}$$

$$x = \frac{-2 \pm i\,2\sqrt{3}}{2}$$

$$x = \cancel{2}\left(\frac{-1 \pm i\sqrt{3}}{\cancel{2}}\right)$$

$$x = -1 \pm i\sqrt{3}$$

We know that

$2\omega = -1 + i\sqrt{3}$ and $2\omega^2 = -1 - i\sqrt{3}$

$\boxed{x = 2\omega}$ and $\boxed{x = 2\omega^2}$



**Q#1:- Find the cube root of "-27"**

Sol// Let

$$x = (-27)^{1/3}$$

Taking cube on both side

$$[x]^3 = \left[(-27)^{1/3}\right]^3$$

$$x^3 = -27$$

$$x^3 + 27 = 0$$

$$x^3 + 3^3 = 0$$

Applying formula

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$(x)^3+(3)^3=0$$

$$(x+3)\left((x)^2+(x)(3)+(3)^2\right)=0$$

$$(x+3)(x^2+3x+9)=0$$

$$x+3=0$$

$$\boxed{x=-3}$$

$$x^2+3x+9=0$$

Compare with

$$ax^2+bx+c=0$$



$$a=1,\ b=3,\ c=9$$

Apply Quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{9-36}}{2}$$

$$x=\frac{-3\pm\sqrt{-27}}{2}$$

$$x = \frac{-3 \pm \sqrt{-9 \times 3}}{2}$$

$$x = \frac{-3 \pm \sqrt{-1}\sqrt{9}\sqrt{3}}{2}$$

$$x = \frac{-3 \pm i\,3\sqrt{3}}{2}$$

$$x = \frac{3(-1 \pm i\sqrt{3})}{2}$$

$$x = 3\left(\frac{-1 \pm i\sqrt{3}}{2}\right)$$

We know that

$$\omega = \frac{-1 + i\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1 - i\sqrt{3}}{2}$$



$$\boxed{x = 3\omega} \text{ and } \boxed{x = 3\omega^2}$$

**Q#1:- Find the cube roots of "64".**

**Sol//** Let

$$x = (64)^{1/3}$$

Taking cube on both sides

$$[x]^3 = \left[(64)^{1/3}\right]^3$$

$$x^3 = 64$$

$$x^3 - 64 = 0$$

$x^3 - 4^3 = 0$ 

$64 = 4 \times 4 \times 4$

$64 = (4)^3$

Applying formula

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(x)^3 - (4)^3 = 0$

$(x-4)\left((x)^2 + (x)(4) + (4)^2\right) = 0$

$(x-4)(x^2 + 4x + 16) = 0$

$x - 4 = 0$

$\boxed{x = 4}$

$x^2 + 4x + 16 = 0$

Compare with

$ax^2 + bx + c = 0$



$a = 1, \quad b = 4, \quad c = 16$

Apply Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$x = \frac{-4 \pm \sqrt{-48}}{2}$$

$$x = \frac{-4 \pm \sqrt{-16 \times 3}}{2}$$

$$x = \frac{-4 \pm \sqrt{-1}\sqrt{16}\sqrt{3}}{2}$$

$$x = \frac{-4 \pm i4\sqrt{3}}{2}$$

$$x = 4\left(\frac{-1 \pm i\sqrt{3}}{2}\right)$$

We know that

$$\omega = \frac{-1 + i\sqrt{3}}{2} \quad \text{and} \quad \omega^2 = \frac{-1 - i\sqrt{3}}{2}$$

$$\boxed{x = 4\omega} \quad \text{and} \quad \boxed{x = 4\omega^2}$$



**Question #2 (II) Evaluate**

$$(1 - 3\omega - 3\omega^2)^5$$

Sol//

$$(1 - 3\omega - 3\omega^2)^5$$

$$= \left[1 - 3(\omega + \omega^2)\right]^5$$

We know that

$$1 + \omega + \omega^2 = 0$$

$$\boxed{\omega + \omega^2 = -1}$$

$$= \left[1 - 3(-1)\right]^5$$

$$= [1 + 3]^5$$

$$= [4]^5$$

$$= \boxed{1024}$$ Answer.

Q#2(viii) $\omega^{-13} + \omega^{-17}$

Sol//

$$\omega^{-13} + \omega^{-17}$$

$$= \frac{1}{\omega^{13}} + \frac{1}{\omega^{17}}$$

$$= \frac{1}{\omega^{12}\cdot\omega} + \frac{1}{\omega^{15}\,\omega^{2}}$$

$$= \frac{1}{(\omega^3)^4\cdot\omega} + \frac{1}{(\omega^3)^5\cdot\omega^2}$$

put $\omega^3 = 1$

$$= \frac{1}{(1)^4\cdot\omega} + \frac{1}{(1)^5\cdot\omega^2}$$



$$= \frac{1}{1\cdot\omega} + \frac{1}{1\cdot\omega^2}$$

$\because (1)^4 = 1$
and
$(1)^5 = 1$

$$= \frac{1}{\omega} + \frac{1}{\omega^2}$$

$$= \frac{\omega + 1}{\omega^2}$$

we know that

$1 + \omega + \omega^2 = 0$

$\boxed{1+\omega = -\omega^2}$

$$= \frac{-\cancel{\omega^2}}{\cancel{\omega^2}}$$

$= \boxed{-1}$ Answer

**Question # 3:- Prove that**

$$x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$$

Sol//

$$R.H.S=(x+y)(x+\omega y)(x+\omega^2 y)$$

$$=(x+y)[x^2+\omega^2 xy+\omega xy+\omega^3 y^2]$$

$$=(x+y)[x^2+xy(\omega^2+\omega)+(\omega^3)y^2]$$

put $\omega^2+\omega=-1$ and $\omega^3=1$

$$=(x+y)[x^2+xy(-1)+(1)y^2]$$

$$=(x+y)(x^2-xy+y^2)$$



$$=x^3+y^3$$

$$=L.H.S$$ Hence proved.

**Question # 4:- Prove that**

$$x^3+y^3+z^3-3xyz=(x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$$

Sol//

$$R.H.S=(x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$$

$$=(x+y+z)[x^2+\omega^2 xy+\omega xz+\omega xy+\omega^3 y^2+\omega^2 yz+\omega^2 xz+\omega^4 yz+\omega^3 z^2]$$

$$= (x+y+z)\left[x^2 + \omega^3 y^2 + \omega^3 z^2 + \omega^2 xy + \omega xy + \omega^2 yz + \omega^4 yz + \omega^2 xz + \omega xz\right]$$

put $\boxed{\omega^3 = 1}$

$$\omega^4 = \omega^3 \cdot \omega = (1)\cdot \omega = \omega$$

$$\boxed{\omega^4 = \omega}$$

$$= (x+y+z)\left[x^2 + (1)y^2 + (1)z^2 + xy(\omega^2+\omega) + yz(\omega^2+\omega^4) + xz(\omega^2+\omega)\right]$$

$$= (x+y+z)\left[x^2+y^2+z^2 + xy(\omega^2+\omega) + yz(\omega^2+\omega) + xz(\omega^2+\omega)\right]$$



put $\boxed{\omega^2 + \omega = -1}$

$$= (x+y+z)\left[x^2+y^2+z^2 + xy(-1) + yz(-1) + xz(-1)\right]$$

$$= (x+y+z)\left[x^2+y^2+z^2 - xy - yz - xz\right]$$

$$= x^3 + \cancel{xy^2} + \cancel{xz^2} - \cancel{x^2y} - xyz - \cancel{x^2z}$$
$$+ \cancel{x^2y} + y^3 + \cancel{yz^2} - \cancel{xy^2} - \cancel{y^2z} - xyz$$
$$+ \cancel{x^2z} + \cancel{y^2z} + z^3 - xyz - \cancel{yz^2} - \cancel{xz^2}$$

$$= x^3 + y^3 + z^3 - 3xyz$$

= R.H.S

Hence proved

# EXERCISE # 2.3

Q#1 (i , v , vi) , Q#2 (ii)

Q#5 (ii) Q#6 (i)

## Question # 1 :-

Without solving, find the sum and the product of the roots of the following quadratic equations.

(i) $x^2 - 5x + 3 = 0$

Sol//

$$1x^2 - 5x + 3 = 0$$
$$ax^2 + bx + c = 0$$

$a = 1$ , $b = -5$ , $c = 3$



$$\text{Sum} = \alpha + \beta = \frac{-b}{a}$$

$$S = \frac{-(-5)}{1}$$

$$\boxed{S = 5}$$

$$\text{Product} = \alpha\beta = \frac{c}{a}$$

$$P = \frac{3}{1}$$

$$\boxed{P = 3}$$ Answer (i).

## Question # 1 (v) $(l+m)x^2 + (m+n)x + n - l = 0$

Sol//

$$(l+m)x^2 + (m+n)x + (n-l) = 0$$
$$ax^2 + bx + c = 0$$

$a = l+m \quad , \quad b = m+n \quad , \quad c = n-l$

$$Sum = \alpha+\beta = -\frac{b}{a}$$

$$\boxed{S = -\frac{(m+n)}{(l+m)}}$$

$$Product = \alpha\beta = \frac{c}{a}$$

$$\boxed{P = \frac{n-l}{l+m}}$$

Answer

## Question #1 (vi)

$$7x^2 - 5mx + 9n = 0$$

Sol//



$$7x^2 - 5mx + 9n = 0$$

$$ax^2 + bx + c = 0$$

$a = 7 \quad , \quad b = -5m \quad , \quad c = 9n$

$$Sum = \alpha+\beta = -\frac{b}{a}$$

$$S = -\frac{(-5m)}{7}$$

$$\boxed{S = \frac{5m}{7}}$$

$$Product = \alpha\beta = \frac{c}{a}$$

$$\boxed{P = \frac{9n}{7}}$$

Answer

## Question # 2(ii)

Find the value of K, if sum of the equation

$$x^2 + (3K-7)x + 5K = 0$$

is $\frac{3}{2}$ times the product of the roots.

Sol//

$$1x^2 + (3K-7)x + 5K = 0$$

$$ax^2 + bx + c = 0$$

$$a = 1, \quad b = 3K-7, \quad c = 5K$$

$$\text{Sum} = -\frac{b}{a}$$

$$S = -\frac{(3K-7)}{1}$$

$$\boxed{S = -3K+7}$$



$$\text{Product} = \frac{c}{a}$$

$$P = \frac{5K}{1}$$

$$\boxed{P = 5K}$$

According to Given Condition

$$\text{Sum} = \frac{3}{2}\text{ product}$$

$$S = \frac{3}{2}P$$

$$-3K+7 = \frac{3}{2}(5K)$$

$$2(-3K+7) = 3(5K)$$

$$-6K + 14 = 15K$$

$$14 = 15K + 6K$$

$$14 = 21K$$

$$\frac{14}{21} = K$$

Divided by "7"

$$\boxed{\frac{2}{3} = K}$$ Answer Sir.

## Question # 5 (ii)

Find "m" if the roots of the equation



$$x^2 + 7x + 3m - 5 = 0$$

satisfy the equation or Relation

$$3\alpha - 2\beta = 4$$

Sol//

$$1x^2 + 7x + 3m - 5 = 0$$
$$ax^2 + bx + c = 0$$

$$a = 1, \quad b = 7, \quad c = 3m - 5$$

$$S = \alpha + \beta = -\frac{b}{a} \quad ; \quad P = \alpha\beta = \frac{c}{a}$$

$$S = \alpha + \beta = -\frac{7}{1} \quad ; \quad \alpha\beta = \frac{3m-5}{1}$$

$$\alpha + \beta = -7 \quad ; \quad \boxed{\alpha\beta = 3m - 5} \longrightarrow (2)$$

$$\boxed{\beta = -7 - \alpha} \longrightarrow (1)$$

Given Relation

$$3\alpha - 2\beta = 4$$

Put the value of "$\beta$" using eq ①

$$3\alpha - 2(-7-\alpha) = 4$$

$$3\alpha + 14 + 2\alpha = 4$$

$$5\alpha = 4 - 14$$

$$5\alpha = -10$$

$$\alpha = -\frac{10}{5}$$

$$\boxed{\alpha = -2}$$

$\beta = ?$

① $\Rightarrow$ $\beta = -7-\alpha$

$$\beta = -7 - (-2)$$

$$\beta = -7 + 2$$

$$\boxed{\beta = -5}$$

② $\Rightarrow$ $\alpha\beta = 3m - 5$

$$(-2)(-5) = 3m - 5$$

put $\alpha = -2$ and $\beta = -5$

$$10 = 3m - 5$$

$$10 + 5 = 3m$$

$$15 = 3m$$

$$\frac{15}{3} = m$$

$$\boxed{5 = m}$$ Answer.

Question #6(i) :-

Find m, if sum and Product of the roots of the following equations is equal to a given number λ

(i) $(2m+3)x^2+(7m-5)x+(3m-10)=0$

Sol/ $(2m+3)x^2+(7m-5)x+(3m-10)=0$

$ax^2+bx+c=0$

$S=\alpha+\beta=-\frac{b}{a}$

$a=2m+3$
$b=7m-5$
$c=3m-10$

$S=\frac{-(7m-5)}{(2m+3)}$

$P=\alpha\beta=\frac{c}{a}$



$P=\frac{3m-10}{(2m+3)}$

According to Given Condition

Sum = Product

$$\frac{-(7m-5)}{\cancel{(2m+3)}}=\frac{3m-10}{\cancel{(2m+3)}}$$

$$-7m+5=3m-10$$

$$5+10=3m+7m$$

$$15=10m$$

$$\frac{15}{10}=m \Rightarrow \boxed{\frac{3}{2}=m} \text{ Answer}$$

# EXERCISE # 2.5

Q#1 (f, g, h) , Q#2 (b, d, e)

Q#3 (b)

**Question #1 :-** Write the quadratic equations having following roots.

(f) $-1 , -7$

Sol//

Roots are $-1 , -7$

Let $\alpha = -1 , \beta = -7$

$S = \alpha + \beta$ $P = \alpha\beta$

$S = (-1) + (-7)$ $P = (-1)(-7)$

$S = -1-7$ $\boxed{P = +7}$



$\boxed{S = -8}$

Put the value of "S" and "P" in the following Quadratic Equation

$$x^2 - Sx + P = 0$$

$$x^2 - (-8)x + (7) = 0$$

$$\boxed{x^2 + 8x + 7 = 0}$$ Answer.

## Q#1 (g)

Sol// $1+i , 1-i$

Let $\alpha = 1+i , \beta = 1-i$

$S = \alpha + \beta$

$S = (1+i) + (1-i)$

$S = 1 + \cancel{i} + 1 - \cancel{i}$

$\boxed{S = 2}$

$P = \alpha\beta$

$P = (1+i)(1-i)$

using formula $(a+b)(a-b) = a^2 - b^2$

$P = (1)^2 - (i)^2$

$P = 1 - i^2$

$P = 1 - (-1) \quad \because i^2 = -1$

$P = 1 + 1$

$\boxed{P = 2}$

put the value of "S" and "P" in the following Quadratic Equation

$x^2 - Sx + P = 0$



$\boxed{x^2 - 2x + 2 = 0}$ Answer.

## Q#1 (h)

$3+\sqrt{2}$ , $3-\sqrt{2}$

Sol:

let $\alpha = 3+\sqrt{2}$ , $\beta = 3-\sqrt{2}$

$S = \alpha + \beta$

$S = (3+\sqrt{2}) + (3-\sqrt{2})$

$S = 3 + \cancel{\sqrt{2}} + 3 - \cancel{\sqrt{2}}$

$\boxed{S = 6}$

$P = \alpha\beta$

$P = (3+\sqrt{2})(3-\sqrt{2})$

using formula

$(a+b)(a-b) = a^2 - b^2$

$P = (3)^2 - (\sqrt{2})^2$

$P = 9 - 2$

$\boxed{P = 7}$

put the value of "S" and "P" in the following Quadratic Equation.

$$x^2 - Sx + P = 0$$

$$\boxed{x^2 - 6x + 7 = 0}$$ Answer.

Question #2 :- If $\alpha, \beta$ are the roots of the equation $x^2 - 3x + 6 = 0$ Form equations whose roots are

(b) $\alpha^2, \beta^2$

Sol//

$$x^2 - 3x + 6 = 0$$
$$ax^2 + bx + c = 0$$

$$a = 1 \ , \ b = -3 \ , \ c = 6$$



$$S = \alpha + \beta = -\frac{b}{a}$$
$$\alpha + \beta = -\frac{(-3)}{1}$$
$$\boxed{\alpha + \beta = 3}$$

$$P = \alpha\beta = \frac{c}{a}$$
$$\alpha\beta = \frac{6}{1}$$
$$\boxed{\alpha\beta = 6}$$

A.T.G.C for $\alpha^2, \beta^2$

$$S = \alpha^2 + \beta^2$$
add and Subtract $2\alpha\beta$
$$S = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$$
$$S = (\alpha + \beta)^2 - 2(\alpha\beta)$$

$$P = \alpha^2\beta^2$$
$$P = (\alpha\beta)^2$$

Now put the value of $\alpha + \beta$ and $\alpha\beta$

$$S = (3)^2 - 2(6)$$

$$P = (6)^2$$

$S = 9 - 12$ | $P = (6)^2$

$S = -3$ | $P = 36$

Required Equation

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 36 = 0$$

$$\boxed{x^2 + 3x + 36 = 0}$$ Answer

Question #2 (d)

$$\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$$

Sol//



$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

Add & Subtract $2\alpha\beta$

$$S = \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$P = \left(\frac{\cancel{\alpha}}{\cancel{\beta}}\right)\left(\frac{\cancel{\beta}}{\cancel{\alpha}}\right)$$

$$\boxed{P = 1}$$

Now put the value of $\alpha+\beta$ and $\alpha\beta$

Already Solved in previous Question

$\alpha + \beta = 3$ , $\alpha\beta = 6$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9-12}{6}$$

$$S = -\frac{3}{6}$$

$$\boxed{S = -\frac{1}{2}}$$

**Required Equation**

$$x^2 - Sx + P = 0$$

$$x^2 - \left(-\frac{1}{2}\right)x + (1) = 0$$

$$x^2 + \frac{x}{2} + 1 = 0$$

Multiply by "2" on both sides



$$2(x^2) + \cancel{2}\left(\frac{x}{\cancel{2}}\right) + 2(1) = 2(0)$$

$$\boxed{2x^2 + x + 2 = 0}$$ Answer.

**Question # 2 (e)**

$$\alpha + \beta \ , \ \frac{1}{\alpha} + \frac{1}{\beta}$$

Sol//

$$S = (\alpha+\beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \qquad P = (\alpha+\beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$S = (\alpha+\beta) + \left(\frac{\beta+\alpha}{\alpha\beta}\right) \qquad P = (\alpha+\beta)\left(\frac{\beta+\alpha}{\alpha\beta}\right)$$

put $\boxed{\alpha+\beta = 3}$ and $\boxed{\alpha\beta = 6}$ (already solved)

$$S = (3) + \left(\frac{3}{6}\right) \qquad P = (3)\left(\frac{3}{6}\right)$$

$S = \frac{3}{1} + \frac{3}{6}$

$P = (\cancel{3})\left(\frac{3}{\cancel{6}_2}\right)$

$S = \frac{18+3}{6}$

$P = (1)\left(\frac{3}{2}\right)$

$S = \frac{21}{6}$

$\boxed{P = \frac{3}{2}}$

Divided by 3

$\boxed{S = \frac{7}{2}}$

Required Equation

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiply by 2 on both sides.



$$2(x^2) - 2\left(\frac{7}{2}x\right) + 2\left(\frac{3}{2}\right) = 2(0)$$

$$\boxed{2x^2 - 7x + 3 = 0}$$ Answer.

**Question #3(b):-** If $\alpha, \beta$ are the roots of the equation

$x^2 + px + q = 0$

Form equations whose roots are

$$\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$$

Sol// First we find $\alpha+\beta$ and $\alpha\beta$

$$1x^2 + px + q = 0$$
$$ax^2 + bx + c = 0$$

$$a = 1 \quad , \quad b = p \quad , \quad c = q$$

$$S = \alpha+\beta = -\frac{b}{a}$$
$$\alpha+\beta = -\frac{p}{1}$$
$$\boxed{\alpha+\beta = -p}$$

$$P = \alpha\beta = \frac{c}{a}$$
$$\alpha\beta = \frac{q}{1}$$
$$\boxed{\alpha\beta = q}$$

According to Given Conditon, Roots are

$$\frac{\alpha}{\beta} \quad , \quad \frac{\beta}{\alpha}$$



$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$S = \frac{\alpha^2+\beta^2}{\alpha\beta}$$

Add and Subtract $2\alpha\beta$

$$S = \frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(\alpha+\beta)^2-2(\alpha\beta)}{\alpha\beta}$$

$$P = \left(\frac{\cancel{\alpha}}{\cancel{\beta}}\right)\left(\frac{\cancel{\beta}}{\cancel{\alpha}}\right)$$

$$\boxed{P = 1}$$

Now put the value of $\alpha+\beta$ and $\alpha\beta$

$$S = \frac{(-p)^2-2q}{q}$$

$$\boxed{S = \frac{p^2 - 2q}{q}}$$

Required Equation

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + (1) = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

Multiply by "q" on both sides.

$$q(x^2) - q\left(\frac{p^2 - 2q}{q}\right)x + q(1) = q(0)$$



$$\boxed{qx^2 - (p^2 - 2q)x + q = 0}$$

Answer.

# EXERCISE # 2.6

## Q#1 , Q#2 , Q#5

**Question #1:-** Use synthetic division to find the quotient and the remainder, when

(i) $(x^2+7x-1) \div (x+1)$

Sol//

put $x+1=0$

$x=-1$

$1x^2+7x-1=0$

$$\begin{array}{r|rrr} -1 & 1 & 7 & -1 \\ & \downarrow & -1 & -6 \\ \hline & 1 & 6 & -7 \end{array}$$

add

multiply

multiply



Quotient $=1x+6$

Remainder $=-7$

**Question #1 (ii)**

$$(4x^3-5x+15) \div (x+3)$$

Sol//

put $x+3=0$

$x=-3$

$4x^3+0x^2-5x+15=0$

$$\begin{array}{r|rrr|r} -3 & 4 & 0 & -5 & 15 \\ & \downarrow & -12 & 36 & -93 \\ \hline & 4 & -12 & 31 & -78 \end{array}$$

Quotient $=4x^2-12x+31$

Remainder $=-78$

## Question #1 (iii)

$$(x^3+x^2-3x+2)\div(x-2)$$

Sol//

put $x-2=0$

$x=2$

$$1x^3+1x^2-3x+2=0$$

$$\begin{array}{c|cccc}
2 & 1 & 1 & -3 & 2 \\
 & \downarrow & 2 & 6 & 6 \\
\hline
 & 1 & 3 & 3 & \boxed{8}
\end{array}$$

$\boxed{\text{Quotient} = 1x^2+3x+3}$



$\boxed{\text{Remainder} = 8}$

Answer

## Question #2 :- Find the value of "h" using synthetic division, if

(i) 3 is the zero of the polynomial $2x^3-3hx^2+9$

Sol//

$$2x^3-3hx^2+0x+9$$

$$\begin{array}{c|cccc}
3 & 2 & -3h & 0 & 9 \\
 & \downarrow & 6 & -9h+18 & -27h+54 \\
\hline
 & 2 & -3h+6 & -9h+18 & \boxed{-27h+63}
\end{array}$$

put Remainder = 0

$$-27h + 63 = 0$$

$$63 = 27h$$

$$\frac{63}{27} = h$$

Divided by "9"

$$\boxed{\frac{7}{3} = h}$$ Answer sir.

Question #2 (ii)

1 is the zero of the polynomial

$x^3 - 2hx^2 + 11$

Sol//



$1x^3 - 2hx^2 + 0x + 11$

$$\begin{array}{c|cccc}
 & 1 & -2h & 0 & 11 \\
1 & \downarrow & 1 & -2h+1 & -2h+1 \\
\hline
 & 1 & -2h+1 & -2h+1 & -2h+12
\end{array}$$

put Remainder = 0

$$-2h + 12 = 0$$

$$-2h = -12$$

$$h = \frac{-12}{-2}$$

$$\boxed{h = 6}$$ Answer sir

## Question # 2 (iii)

-1 is the zero of the polynomial

$2x^3 + 5hx - 23$

Sol//

$2x^3 + 0x^2 + 5hx - 23$

$$
\begin{array}{c|cccc}
-1 & 2 & 0 & 5h & -23 \\
 & \downarrow & -2 & 2 & -5h-2 \\
\hline
 & 2 & -2 & 5h+2 & -5h-25
\end{array}
$$

put

Remainder $= 0$



$-5h - 25 = 0$

$-5h = 25$

$h = \frac{25}{-5}$

$\boxed{h = -5}$ Answer

## Question # 5 (i) Solve by using synthetic division, if

(i) 1 and 3 are the roots of the equation

$$x^4 - 10x^2 + 9 = 0$$

Sol//

$$x^4 + 0x^3 - 10x^2 + 0x + 9 = 0$$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | -10 | 0 | 9 |
| | ↓ | 1 | 1 | -9 | -9 |
| | 1 | 1 | -9 | -9 | 0 |
| 3 | ↓ | 3 | 12 | 9 | |
| | 1 | 4 | 3 | 0 | |

$$1x^2 + 4x + 3 = 0$$



Solve by factorization

$3x^2$

$+1x$ $+3x$

$$1x^2 + 1x + 3x + 3 = 0$$

$$x(x+1) + 3(x+1) = 0$$

$$(x+1)(x+3) = 0$$

$x + 1 = 0$ | $x + 3 = 0$

$\boxed{x = -1}$ | $\boxed{x = -3}$

**$-1, -3, 1$ and $3$**

are the roots of the equation.

**Question #5 (ii)** 3 and $-4$ are the roots of the equation

$$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$$

Sol//

$$1x^4 + 2x^3 - 13x^2 - 14x + 24$$

$$\begin{array}{r|rrrr|r}
3 & 1 & 2 & -13 & -14 & 24 \\
 & \downarrow & 3 & 15 & 6 & -24 \\
\hline
 & 1 & 5 & 2 & -8 & 0 \\
-4 & \downarrow & -4 & -4 & 8 & \\
\hline
 & 1 & 1 & -2 & 0 &
\end{array}$$



$$1x^2 + 1x - 2 = 0$$

$$1x^2 - 1x + 2x - 2 = 0$$

$-2x^2$

$-1x \quad +2x$

$$x(x-1) + 2(x-1) = 0$$

$$(x-1)(x+2) = 0$$

| | |
|---|---|
| $x - 1 = 0$ | $x + 2 = 0$ |
| $\boxed{x = 1}$ | $\boxed{x = -2}$ |

$1, -2, 3$ and $-4$ are the roots of the equation.

# EXERCISE # 2.7

Question # 2, 5, 10, 13

Solve the following simultaneous equations.

## Question # 2 :-

$$3x - 2y = 1 \quad ; \quad x^2 + xy - y^2 = 1$$

Sol/

$3x - 2y = 1 \longrightarrow (1)$

$x^2 + xy - y^2 = 1 \longrightarrow (2)$

$3x = 1 + 2y$

$x = \dfrac{1+2y}{3} \longrightarrow (3)$



put the value of "$x$" in equation # 2

$$x^2 + xy - y^2 = 1$$

$$\left(\frac{1+2y}{3}\right)^2 + \left(\frac{1+2y}{3}\right)y - y^2 = 1$$

$$\frac{(1+2y)^2}{(3)^2} + \frac{(1+2y)y}{3} - y^2 = 1$$

$$\frac{(1)^2 + (2y)^2 + 2(1)(2y)}{9} + \frac{y + 2y^2}{3} - y^2 = 1$$

$$\frac{1 + 4y^2 + 4y}{9} + \frac{y + 2y^2}{3} - y^2 = 1$$

Multiply by "9" on both sides

$$9\left(\frac{1+4y^2+4y}{9}\right)+\overset{3}{\cancel{9}}\left(\frac{y+2y^2}{\cancel{3}}\right)-9(y^2)=9(1)$$

$$1+4y^2+4y+3(y+2y^2)-9y^2=9$$

$$1+4y^2+4y+3y+6y^2-9y^2=9$$

$$y^2+7y+1-9=0$$

$$y^2+7y-8=0$$

$$y^2-1y+8y-8=0$$

$-8y^2$

$-1y \quad +8y$



$$y(y-1)+8(y-1)=0$$

$$(y-1)(y+8)=0$$

| | |
|---|---|
| $y-1=0$ | $y+8=0$ |
| $y=1$ | $y=-8$ |
| Now $x=?$ | Now $x=?$ |
| Put the value of "y" in equation # 3 | put the value of "y" in equation # 3 |
| $x=\frac{1+2y}{3}$ | $x=\frac{1+2y}{3}$ |
| $x=\frac{1+2(1)}{3}$ | $x=\frac{1+2(-8)}{3}$ |

$x = \frac{1+2}{3}$ | $x = \frac{1-16}{3}$

$x = \frac{3}{3}$ | $x = -\frac{15}{3}$

$x = 1$ | $x = -5$

$(x, y) = (1, 1)$ | $(x, y) = (-5, -8)$

## Question#5:-

$$x^2 + (y-1)^2 = 10 \quad ; \quad x^2 + y^2 + 4x = 1$$

Sol//

$x^2 + (y-1)^2 = 10 \longrightarrow (1)$ | $x^2 + y^2 + 4x = 1 \longrightarrow (2)$



$$x^2 + \left[(y)^2 + (1)^2 - 2(y)(1)\right] = 10$$

$$x^2 + y^2 + 1 - 2y = 10$$

$$x^2 + y^2 - 2y + 1 - 10 = 0$$

$$x^2 + y^2 - 2y - 9 = 0$$

$$x^2 + y^2 - 2y = 9 \longrightarrow (3)$$

Subtract equation #(2) from eq (3)

$$\cancel{x^2} + \cancel{y^2} - 2y = 9$$

$$\underline{\underset{-}{\cancel{x^2}} \underset{-}{+} \cancel{y^2} \underset{-}{+4x} = \underset{-}{1}}$$

$$-2y - 4x = 8$$

$$-4x = 8+2y$$

$$\boxed{x = \frac{8+2y}{-4}} \longrightarrow (4)$$

put the value of "x" in eq (2)

$$x^2 + y^2 + 4x = 1$$

$$\left(\frac{8+2y}{-4}\right)^2 + y^2 + \cancel{4}\left(\frac{8+2y}{\cancel{-4}}\right) = 1$$

$$\frac{(8+2y)^2}{(-4)^2} + y^2 - (8+2y) = 1$$

$$\frac{(8+2y)^2}{16} + y^2 - 8 - 2y = 1$$



$$\frac{(8)^2 + (2y)^2 + 2(8)(2y)}{16} + y^2 - 8 - 2y = 1$$

$$\frac{64 + 4y^2 + 32y}{16} + \frac{y^2 - 2y}{1} = 1 + 8$$

Multiply by "16" on both sides

$$\cancel{16}\left(\frac{64+4y^2+32y}{\cancel{16}}\right) + 16\left(\frac{y^2-2y}{1}\right) = 16(9)$$

$$64 + 4y^2 + \cancel{32y} + 16y^2 - \cancel{32y} = 144$$

$$20y^2 = 144 - 64$$

$$20y^2 = 80$$
$$y^2 = \frac{80}{20}$$
$$y^2 = 4$$

Taking square root on both side

$$\sqrt{y^2} = \sqrt{4}$$
$$y = \pm 2$$

put the value of y in eq (4)

If $\boxed{y = 2}$

$$x = \frac{8+2y}{-4}$$
$$x = \frac{8+2(2)}{-4}$$
$$x = \frac{8+4}{-4}$$
$$x = \frac{12}{-4}$$
$$\boxed{x = -3}$$

If $\boxed{y = -2}$

$$x = \frac{8+2y}{-4}$$
$$x = \frac{8+2(-2)}{-4}$$
$$x = \frac{8-4}{-4}$$
$$x = \frac{4}{-4}$$
$$\boxed{x = -1}$$

$(x, y) = (-3, 2)$

$(x, y) = (-1, -2)$

## Question # 10 :-

$$x^2+2y^2=3 \quad ; \quad x^2+4xy-5y^2=0$$

Sol//

$$x^2+2y^2=3 \longrightarrow (1) \qquad x^2+4xy-5y^2=0 \longrightarrow (2)$$

Solve eq (2) by factorization

$$x^2+4xy-5y^2=0$$

$(x^2)(-5y^2)$

$-5x^2y^2$

$-1xy \quad +5xy$

$$x^2-1xy+5xy-5y^2=0$$

$$x(x-y)+5y(x-y)=0$$

$$\therefore (x-y)(x+5y)=0$$



| | |
|---|---|
| $x-y=0$ | $x+5y=0$ |
| $x=y \longrightarrow (3)$ | $x=-5y \longrightarrow (4)$ |
| put the value of "$x$" in eq, # (1) | put the value of "$x$" in eq, # (1) |
| $x^2+2y^2=3$ | $x^2+2y^2=3$ |
| $(y)^2+2y^2=3$ | $(-5y)^2+2y^2=3$ |
| $y^2+2y^2=3$ | $25y^2+2y^2=3$ |
| $3y^2=3$ | $27y^2=3$ |
| $y^2=\frac{3}{3}$ | $y^2=\frac{3}{27}$ |
| $y^2=1$ | $y^2=\frac{1}{9}$ |

Taking Square Root on both sides

$\sqrt{y^2} = \sqrt{1}$

$\boxed{y = \pm 1}$

Now put the value of "y" in eq (3)

$x = y$

$\boxed{x = \pm 1}$

$(x, y) = (\pm 1, \pm 1)$

$\sqrt{y^2} = \sqrt{\frac{1}{9}}$

$\boxed{y = \pm \frac{1}{3}}$

Put the value of "y" in eq (4)

$x = -5y$

$x = -5\left(\pm \frac{1}{3}\right)$

$\boxed{x = \mp \frac{5}{3}}$

$(x, y) = \left(\mp \frac{5}{3}, \pm \frac{1}{3}\right)$



## Question #13 :-

$x^2 - 2xy = 7 \quad ; \quad xy + 3y^2 = 2$

Sol//

$x^2 - 2xy = 7 \rightarrow (1)$

Multiply eq (1) by "2"

$2(x^2 - 2xy) = 2(7)$

$2x^2 - 4xy = 14 \rightarrow (3)$

$xy + 3y^2 = 2 \rightarrow (2)$

Multiply eq (2) by "7"

$7(xy + 3y^2) = 7(2)$

$7xy + 21y^2 = 14 \rightarrow (4)$

Subtract eq (4) from eq (3)

$$2x^2 - 4xy \qquad = 14$$
$$\underline{\phantom{2x^2}\;7xy + 21y^2 = 14}$$
$$- \qquad - \qquad -$$

$$2x^2 - 11xy + 21y^2 = 0$$

Now solve by factorization

$$(2x^2)(21y^2)$$
$$+42x^2y^2$$
$$+3xy \qquad -14xy$$

$$2x^2 - 11xy + 21y^2 = 0$$
$$2x^2 + 3xy - 14xy + 21y^2 = 0$$
$$x(2x+3y) - 7y(2x+3y) = 0$$

$$(2x+3y)(x-7y) = 0$$



$$2x + 3y = 0 \qquad x - 7y = 0$$

$$2x = -3y$$
$$x = -\frac{3}{2}y \rightarrow ⑤$$

put the value of "$x$" in eq ①

$$x^2 - 2xy = 7$$
$$\left(-\frac{3}{2}y\right)^2 - 2\left(-\frac{3}{2}y\right)y = 7$$

$$\frac{9y^2}{4} + 3y^2 = 7$$

Multiply by "4" on both sides

$$x = 7y \longrightarrow ⑥$$

put the value of "$x$" in eq ①

$$x^2 - 2xy = 7$$
$$(7y)^2 - 2(7y)y = 7$$
$$49y^2 - 14y^2 = 7$$

Divided by 7

$$\frac{49y^2}{7} - \frac{14y^2}{7} = \frac{7}{7}$$

$$\cancel{4}\left(\frac{9y^2}{\cancel{4}}\right) + 4(3y^2) = 4(7)$$

$$9y^2 + 12y^2 = 28$$

$$21y^2 = 28$$

$$y^2 = \frac{28}{21}$$

$$y^2 = \frac{4}{3}$$

Taking Square Root on both sides

$$\sqrt{y^2} = \sqrt{\frac{4}{3}}$$

$$\boxed{y = \pm \frac{2}{\sqrt{3}}}$$

Put in eq 5

$$x = -\frac{3}{2}y$$

$$x = -\frac{3}{\cancel{2}}\left(\pm \frac{\cancel{2}}{\sqrt{3}}\right)$$

$$x = \mp \frac{3}{\sqrt{3}}$$

$$x = \mp \frac{(\sqrt{3})^{\cancel{2}}}{\cancel{\sqrt{3}}}$$

$$\boxed{x = \mp\sqrt{3}}$$

$$(x, y) = \left(\mp\sqrt{3}, \pm \frac{2}{\sqrt{3}}\right)$$

$$7y^2 - 2y^2 = 1$$

$$5y^2 = 1$$

$$y^2 = \frac{1}{5}$$

Taking Square Root on both sides

$$\sqrt{y^2} = \sqrt{\frac{1}{5}}$$

$$\boxed{y = \pm \frac{1}{\sqrt{5}}}$$

Put in eq 6

$$x = 7y$$

$$x = 7\left(\pm \frac{1}{\sqrt{5}}\right)$$

$$\boxed{x = \pm \frac{7}{\sqrt{5}}}$$

$$(x, y) = \left(\pm \frac{7}{\sqrt{5}}, \pm \frac{1}{\sqrt{5}}\right)$$

Answer

# EXERCISE #2.8

## Question # 1 , 4 , 5 , 9 , 10

**Question #1 :-**

The Product of two positive consecutive numbers is 182. Find the numbers.

**Sol/**

let 1st number $= x$

2nd number $= x+1$

According to given condition

$$(x)(x+1) = 182$$

$$x^2 + x = 182$$

$$x^2 + x - 182 = 0$$



$-182x^2$

| | | |
|---|---|---|
| 1 | 182 | X |
| 2 | 91 | X |
| $-13x$ | $+14x$ | ✓ |

$$x^2 - 13x + 14x - 182 = 0$$

$$x(x-13) + 14(x-13) = 0$$

$$(x-13)(x+14) = 0$$

$x - 13 = 0$

$\boxed{x = 13}$

1st number $= x = \boxed{13}$

2nd number $= x+1 = 13+1$

$= \boxed{14}$

$x + 14 = 0$

$\boxed{x = -14}$

Neglected because it is negative number

## Question # 4 :-

The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

Sol

Let number $= x$

According to Given Condition

$$(3x-5)(4x-1) = 7$$

$$12x^2 - 3x - 20x + 5 = 7$$

$$12x^2 - 23x + 5 - 7 = 0$$



$$12x^2 - 23x - 2 = 0$$

Solve by factorization

$(12x^2)(-2)$

$-24x^2$

$+1x \quad -24x$

$$12x^2 + 1x - 24x - 2 = 0$$

$$x(12x+1) - 2(12x+1) = 0$$

$$(12x+1)(x-2) = 0$$

$12x + 1 = 0$

$12x = -1$

$\boxed{x = -\frac{1}{2}}$

$x - 2 = 0$

$\boxed{x = 2}$

Answer

Question#5:-

The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Sol/

Let number $= x$

its reciprocal $= \frac{1}{x}$

According to Given Condition

$$x - \frac{1}{x} = \frac{15}{4}$$

Multiply by "x" on both sides

$$x(x) - x\left(\frac{1}{x}\right) = x\left(\frac{15}{4}\right)$$

$$x^2 - 1 = \frac{15x}{4}$$



$$4(x^2 - 1) = 15x$$

$$4x^2 - 4 = 15x$$

$$4x^2 - 15x - 4 = 0$$

Solve by factorization

$(4x^2)(-4)$

$-16x^2$

$+1x \quad -16x$

$$4x^2 + 1x - 16x - 4 = 0$$

$$x(4x+1) - 4(4x+1) = 0$$

$$(4x+1)(x-4) = 0$$

| | |
|---|---|
| $4x+1=0$ | $x-4=0$ |
| $4x=-1$ | $\boxed{x=4}$ |
| $\boxed{x=-\frac{1}{4}}$ | |

Answer

Question#9 :-

Find two integers whose difference is 4 and whose squares differ by 72.

Sol

let 1st integer = $x$

2nd integer = $y$

According to Given Condition

$$x - y = 4 \longrightarrow (1)$$

$$x^2 - y^2 = 72 \longrightarrow (2)$$

(1) $\Rightarrow$ $x - y = 4$



$$x = 4 + y$$

Put the value of "$x$" in eq (2)

$$(4+y)^2 - y^2 = 72$$

Using formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$(4)^2 + (y)^2 + 2(4)(y) - y^2 = 72$$

$$16 + \cancel{y^2} + 8y - \cancel{y^2} = 72$$

$$8y = 72 - 16$$

$$8y = 56$$

$$y = \frac{56}{8}$$

$\boxed{y = 7}$

put the value of "y" in eq ①

$$x - y = 4$$

$$x - 7 = 4$$

$$x = 4 + 7$$

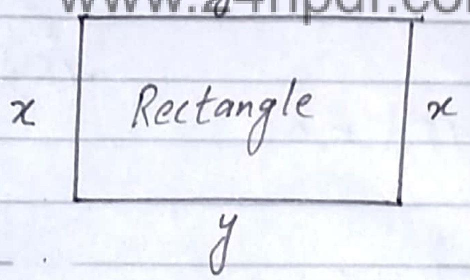
$$\boxed{x = 11}$$ Answer.

## Question #10:-

Find the dimensions of a rectangle, whose perimeter is 80 cm and its area is 375 $cm^2$.

Sol






$$\text{Perimeter} = 2(x + y)$$

$$80 = 2(x + y)$$

$$\frac{80}{2} = x + y$$

$$40 = x + y \longrightarrow ①$$

$$\text{Area} = xy$$

$$375 = xy \longrightarrow ②$$

① $\Rightarrow$ $40 = x + y$

$$\boxed{40 - y = x} \longrightarrow ③$$

put the value of "x" in eq ②

$375 = xy$

$375 = (40 - y)y$

$375 = 40y - y^2$

$y^2 - 40y + 375 = 0$

Solve by factorization

$+375y^2$

$-15y \quad -25y$



$y^2 - 15y - 25y + 375 = 0$

$y(y-15) - 25(y-15) = 0$

$(y-15)(y-25) = 0$

$y - 15 = 0$

$\boxed{y = 15}$

Put the value of "y" in eq ③

$40 - y = x$

$40 - 15 = x$

$\boxed{25 = x}$

$y - 25 = 0$

$\boxed{y = 25}$

Put the value of "y" in eq ③

$40 - y = x$

$40 - 25 = x$

$\boxed{15 = x}$

# Miscellaneous EXERCISE 2

**Q#2 (i — vii)** Write short answers of the following question.

**Q#2(i):-** Discuss the nature of the roots of the following equations.

Sol//

**(a)** $x^2+3x+5=0$

$ax^2+bx+c=0$

$a=1, \quad b=3, \quad c=5$

formula

$$\text{Disc.} = b^2-4ac$$

$$\text{Disc.} = (3)^2-4(1)(5)$$

$$\text{Disc.} = 9-20$$



$$\text{Disc.} = -11 < 0 \quad (-ve)$$

Disc. = is negative

So, roots are imaginary

**Q#1(b)** $2x^2-7x+3=0$

$ax^2+bx+c=0$

$a=2, \quad b=-7, \quad c=3$

formula

$$\text{Disc.} = b^2-4ac$$

$$\text{Disc.} = (-7)^2-4(2)(3)$$

$$\text{Disc.} = 49-24$$

$$\text{Disc.} = 25$$

Disc. $= 5^2$ (perfect square)

Disc. = Perfect square,

So, Roots are Rational (Real) and unequal.

**Q#1 (c)**

$$x^2 + 6x - 1 = 0$$

Sol,

$$ax^2 + bx + c = 0$$

$$a = 1, \quad b = 6, \quad c = -1$$

formula

$$\text{Disc.} = b^2 - 4ac$$

$$\text{Disc.} = (6)^2 - 4(1)(-1)$$



$$\text{Disc.} = 36 + 4$$

$$\text{Disc.} = 40 > 0 \quad (+ve)$$

Disc = +ve and not Perfect square

So, Roots are Irrational (Real) and unequal.

**Q#1 (d) :-**

$$16x^2 - 8x + 1 = 0$$

Sol,

$$ax^2 + bx + c = 0$$

$$a = 16, \quad b = -8, \quad c = 1$$

formula

$$\text{Disc.} = b^2 - 4ac$$

$Disc. = (-8)^2 - 4(16)(1)$

$Disc. = 64 - 64$

$Disc. = 0$

Disc $= 0$, therefore roots are Rational (Real) and equal.

Question#2:–

Find $w^2$

if $w = \dfrac{-1+\sqrt{-3}}{2}$

Sol//

$w = \dfrac{-1+\sqrt{-3}}{2}$



Taking square on both side

$(w)^2 = \left(\dfrac{-1+\sqrt{-3}}{2}\right)^2$

$w^2 = \dfrac{(-1+\sqrt{-3})^2}{(2)^2}$

using formula $(a+b)^2 = a^2+b^2+2ab$

$w^2 = \dfrac{(-1)^2+(\sqrt{-3})^2+2(-1)(\sqrt{-3})}{4}$

$w^2 = \dfrac{1+(-3)-2\sqrt{-3}}{4}$

$w^2 = \dfrac{1-3-2\sqrt{-3}}{4}$

$w^2 = \dfrac{-2-2\sqrt{-3}}{4} \Rightarrow w^2 = \cancel{2}\left(\dfrac{-1-\sqrt{-3}}{\cancel{4}\,2}\right)$

$\boxed{w^2 = \dfrac{-1-\sqrt{-3}}{2}}$ Answer

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Q#2(iii):- Prove that the sum of the all cube roots of unity is zero.

Sol/

We know that cube roots of unity are

$$1, \omega, \omega^2$$

We have to prove that

$$1+\omega+\omega^2 = 0$$

$$L.H.S = 1+\omega+\omega^2$$

$$= 1+\left(\frac{-1+\sqrt{-3}}{2}\right)+\left(\frac{-1-\sqrt{-3}}{2}\right)$$



Taking L.C.M

$$= \frac{2+(-1+\sqrt{-3})+(-1-\sqrt{-3})}{2}$$

$$= \frac{2-1+\cancel{\sqrt{-3}}-1-\cancel{\sqrt{-3}}}{2}$$

$$= \frac{2-2}{2}$$

$$= \frac{0}{2}$$

$$= 0$$

$$= R.H.S$$

Hence proved

**Q#2 (iv):-** Find the product of complex cube roots of unity.

Sol

Complex Cube roots are $\omega$ & $\omega^2$

We have to prove that

$$(\omega)(\omega^2) = 1$$

$$L.H.S = (\omega)(\omega^2)$$

$$= \left(\frac{-1+\sqrt{-3}}{2}\right)\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$= \frac{(-1+\sqrt{-3})(-1-\sqrt{-3})}{4}$$

Using formula $(a+b)(a-b) = (a)^2 - (b)^2$



Let $a = -1$, $b = \sqrt{-3}$

$$= \frac{(-1)^2 - (\sqrt{-3})^2}{4}$$

$$= \frac{1-(-3)}{4}$$

$$= \frac{1+3}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

$$= R.H.S$$

Hence proved.

**Q#2(v):-** Show that

$$x^3+y^3=(x+y)(x+wy)(x+w^2y)$$

Sol/

$$x^3+y^3=(x+y)(x+wy)(x+w^2y)$$

$$R.H.S=(x+y)(x+wy)(x+w^2y)$$

$$=(x+y)\left[x^2+w^2xy+wxy+w^3y^2\right]$$

$$=(x+y)\left[x^2+xy(w^2+w)+(w^3)y^2\right]$$

put

$$w^2+w=-1 \quad \& \quad w^3=1$$



$$=(x+y)\left[x^2+xy(-1)+(1)y^2\right]$$

$$=(x+y)(x^2-xy+y^2)$$

$$=(x)^3+(y)^3$$

$$=x^3+y^3$$

$$= L.H.S$$

Hence proved.

**Q#2(vi):-** Evaluate

$$\omega^{37}+\omega^{38}+1$$

Sol//

$$\omega^{37}+\omega^{38}+1$$

$$= \omega^{36}\cdot\omega^{1} + \omega^{36}\cdot\omega^{2} + 1$$

$$= (\omega^{3})^{12}\cdot\omega + (\omega^{3})^{12}\cdot\omega^{2}+1$$

put $\omega^{3}=1$

$$= (1)^{12}\omega + (1)^{12}\cdot\omega^{2}+1$$

$$= (1)\omega + (1)\omega^{2}+1$$

$$= \omega+\omega^{2}+1$$

$$= \boxed{0}$$ Answer



**Q#2(vii):-** Evaluate $(1-\omega+\omega^{2})^{6}$

Sol//

$$(1-\omega+\omega^{2})^{6}$$

$$= \left[1+\omega^{2}-\omega\right]^{6}$$ Rearrange

$$= \left[(1+\omega^{2})-\omega\right]^{6}$$

We know that

$1+\omega+\omega^{2}=0$

$\because \boxed{1+\omega^{2}=-\omega}$

and

$\boxed{\omega^{3}=1}$

$$= \left[-\omega-\omega\right]^{6}$$

$$= \left[-2\omega\right]^{6}$$

$$= (-2)^{6}\omega^{6}$$

$$= 64(\omega^{3})^{2} = 64(1)^{2} = 64(1) = \boxed{64}$$ Answer

# MATHEMATICS 10th (Science Group)

## Unit # 3

## Variations

### EXERCISE # 3.1

Question # 1(iv), (v)  Question # 4, 5, 7, 9

Question # 11 (iv), (v)

**Question # 1** :- Express the following as a ratio a:b and as a fraction in its simplest (lowest) form.

(iv) 27 min. 30 sec , 1 hour



Sol/

$$27\text{ min. }30\text{ sec} : 1\text{ hour}$$

$$27 \times 60\text{ sec} + 30\text{ sec} : 1 \times 60 \times 60\text{ sec}$$

$$1620\text{ sec} + 30\text{ sec} : 3600\text{ sec}$$

$$1650\text{ sec} : 3600\text{ sec}$$

Divided by 10

$$\frac{1650}{10}\text{ sec} : \frac{3600}{10}\text{ sec}$$

$$165 : 360$$

Divided by 5

$$\frac{165}{5} : \frac{360}{5}$$

$$33 : 72$$

Divided by 3

$\frac{33}{3} : \frac{72}{3}$

$11 : 24$

$= \boxed{\frac{11}{24}}$ Answer

## Question #1 (v)

$75^\circ , 225^\circ$

Sol//

$75^\circ : 225^\circ$

Divided by 5



$\frac{75}{5} : \frac{225}{5}$

$15 : 45$

Divided by 5

$\frac{15}{5} : \frac{45}{5}$

$3 : 9$

Divided by 3

$\frac{3}{3} : \frac{9}{3}$

$1 : 3$

$= \boxed{\frac{1}{3}}$ Answer

**Question # 4:-**

Find the value of "p" if the ratios $2p+5 : 3p+4$ and $3 : 4$ are equal.

Sol// As the given ratios are equal

So

$$2p+5 : 3p+4 = 3 : 4$$

$$\frac{2p+5}{3p+4} = \frac{3}{4}$$

$$4(2p+5) = 3(3p+4)$$

$$8p+20 = 9p+12$$



$$20-12 = 9p-8p$$

$$8 = 1p$$

$$\boxed{8 = p}$$ Answer

**Question # 5**

If the ratio $3x+1 : 6+4x$ and $2:5$ are equal. Find the value of $x$.

Sol// As the given ratios are equal

So

$$3x+1 : 6+4x = 2 : 5$$

$$\frac{3x+1}{6+4x} = \frac{2}{5}$$

$5(3x+1) = 2(6+4x)$

$15x + 5 = 12 + 8x$

$15x - 8x = 12 - 5$

$7x = 7$

$x = \frac{7}{7}$

$\boxed{x = 1}$ Answer

Question #7:- If 10 is added in each number of the ratio 4:13, we get a new ratio 1:2. What are the numbers?

Sol Let the required numbers are $4x$ and $13x$



According to Given Condition

$4x+10 : 13x+10 = 1 : 2$

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$2(4x+10) = 1(13x+10)$

$8x + 20 = 13x + 10$

$20 - 10 = 13x - 8x$

$10 = 5x$

$\frac{10}{5} = x$

$\boxed{2 = x}$

Numbers are

$$4x = 4(2) = \boxed{8}$$

$$13x = 13(2) = \boxed{26} \text{ Answer.}$$

## Question #9:-

If $a:b = 7:6$

find the value of $3a+5b : 7b-5a$

Sol//

Give $a:b = 7:6$

$$\frac{a}{b} = \frac{7}{6}$$

Also

$3a+5b : 7b-5a$



$$= \frac{3a+5b}{7b-5a}$$

Divided by "b"

$$= \frac{\frac{3a+5b}{b}}{\frac{7b-5a}{b}}$$

$$= \frac{3\frac{a}{b} + 5\frac{b}{b}}{7\frac{b}{b} - 5\frac{a}{b}}$$

$$= \frac{3\left(\frac{a}{b}\right)+5}{7-5\left(\frac{a}{b}\right)}$$

put $\frac{a}{b} = \frac{7}{6}$

$$= \frac{3\left(\frac{7}{6}\right)+5}{7-5\left(\frac{7}{6}\right)}$$

$$= \frac{\frac{21}{6}+5}{7-\frac{35}{6}}$$

$$= \frac{\frac{21+30}{6}}{\frac{42-35}{6}}$$

$$= \boxed{\frac{51}{7}} \text{ Answer}$$

Question # 11 (iv)

Find "$x$" in the following proportions.

$$p^2+pq+q^2 : x :: \frac{p^3-q^3}{p+q} : (p-q)^2$$

Sol//

$$p^2+pq+q^2 : x :: \frac{p^3-q^3}{p+q} : (p-q)^2$$

Product of Extremes = Product of means

$$(p^2+pq+q^2)(p-q)^2 = (x)\left(\frac{p^3-q^3}{p+q}\right)$$

$$\frac{(p^2+pq+q^2)(p-q)^2(p+q)}{(p^3-q^3)} = x$$



$$\frac{\cancel{(p^2+pq+q^2)}\cancel{(p-q)}(p-q)(p+q)}{\cancel{(p-q)}\cancel{(p^2+pq+q^2)}} = x$$

$$(p-q)(p+q) = x$$

Using formula

$$(a-b)(a+b) = (a)^2-(b)^2$$

$$(p)^2-(q)^2 = x$$

$$\boxed{p^2-q^2 = x}$$

Answer.

## Question # 11 (v)

$$8-x : 11-x :: 16-x : 25-x$$

Sol/

$$8-x : 11-x :: 16-x : 25-x$$

Product of Extremes = Product of means

$$(8-x)(25-x) = (11-x)(16-x)$$

$$200 - 8x - 25x + x^2 = 176 - 11x - 16x + x^2$$

$$200 - 176 - 33x + \cancel{x^2} - \cancel{x^2} + 11x + 16x = 0$$

$$24 - 33x + 11x + 16x = 0$$



$$24 - 6x = 0$$

$$24 = 6x$$

$$\frac{24}{6} = x$$

$$\boxed{4 = x}$$ Answer

# EXERCISE # 3.2

Q#1 (iii) , Q#2 (ii) , Q#5

Q#8 , Q#10 , Q#11 , Q#13.

**Question # 1 (iii)**

If $y$ varies directly as $x$ and $y = 8$, when $x = 2$ find $x$ when $y = 28$.

Sol

$$y \propto x$$



$$y = Kx \longrightarrow (1)$$

put $y = 8$ and $x = 2$ (Given)

$$8 = K(2)$$

$$\frac{8}{2} = K$$

$$\boxed{4 = K}$$

Put the value of "K" in eq (1)

$$\boxed{y = 4x} \longrightarrow (2)$$

Now $x = ?$ when $y = 28$

put $y = 28$ in eq (2)

$y = 4x$

$28 = 4x$

$\frac{28}{4} = x$

$\boxed{7 = x}$ Answer

Question # 2 (ii)

If $y \propto x$ and $y = 7$ when $x = 3$ find $x$ when $y = 35$ and $y$ when $x = 18$



Sol//

$y \propto x$

$y = Kx \longrightarrow (1)$

put $y = 7$ and $x = 3$ (Given)

$7 = K(3)$

$\boxed{\frac{7}{3} = K}$

Put the value of "K" in eq (1)

$\boxed{y = \frac{7}{3}x} \longrightarrow (2)$

Now $x = ?$ when $y = 35$

put $y = 35$ in eq (2)

$$y = \frac{7}{3}x$$

$$35 = \frac{7}{3}x \qquad \because y = 35$$

$$\frac{\overset{5}{\cancel{35}} \times 3}{\cancel{7}_1} = x$$

$\boxed{15 = x}$ Answer.

Now $y = ?$ when $x = 18$

put $x = 18$ in eq ②

$$y = \frac{7}{3}x$$

$$y = \frac{7}{3}(18) \qquad \because x = 18$$



$$y = \frac{7}{\cancel{3}_1}(\overset{6}{\cancel{18}})$$

$\boxed{y = 42}$ Answer.

**Question#5 :-** If $V \propto R^3$ and $V = 5$ when $R = 3$, Find $R$ when $V = 625$.

Sol//

$$V \propto R^3$$

$$V = KR^3 \longrightarrow ①$$

put $V = 5$ and $R = 3$ in eq ①

$5 = K(3)^3$

$5 = K(27)$

$\boxed{\frac{5}{27} = K}$

put the value of "K" in eq ①

$\boxed{V = \frac{5}{27} R^3} \longrightarrow ②$

Now

$R = ?$ when $V = 625$

put $V = 625$ in eq ②

$V = \frac{5}{27} R^3$

$625 = \frac{5}{27} R^3$



$\frac{625 \times 27}{5} = R^3$

$125 \times 27 = R^3$

$5^3 \times 3^3 = R^3$

$(5 \times 3)^3 = R^3$

Taking cube root on both side

$[(5 \times 3)^3]^{1/3} = [R^3]^{1/3}$

$5 \times 3 = R$

$\boxed{15 = R}$ Answer sir.

Question #8:- If $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find $x$ when $y = 24$.

Sol

$$y \propto \frac{1}{x}$$

$$y = K\frac{1}{x} \longrightarrow (1)$$

put $y = 4$ and $x = 3$ (Given)

$$4 = K\left(\frac{1}{3}\right)$$

$$4 \times 3 = K$$

$$\boxed{12 = K}$$

put the value of "K" in eq (1)



$$y = 12\left(\frac{1}{x}\right)$$

$$\boxed{y = \frac{12}{x}} \longrightarrow (2)$$

Now

$x = ?$ when $y = 24$

put $y = 24$ in eq (2)

$$y = \frac{12}{x}$$

$$24 = \frac{12}{x}$$

$$24x = 12$$

$$x = \frac{12}{24} \Rightarrow \boxed{x = \frac{1}{2}}$$ Answer

Question #10:- $A \propto \frac{1}{r^2}$ and $A = 2$ when $r = 3$, find $r$ when $A = 72$.

Sol//

$$A \propto \frac{1}{r^2}$$

$$A = K\frac{1}{r^2}$$

$$A = \frac{K}{r^2} \longrightarrow (1)$$

put $A = 2$ and $r = 3$ (Given)

$$2 = \frac{K}{(3)^2}$$

$$2 = \frac{K}{9}$$

$$2 \times 9 = K$$



$$\boxed{18 = K}$$

put the value of "K" in eq (1)

$$\boxed{A = \frac{18}{r^2}} \longrightarrow (2)$$

Now

$r = ?$ when $A = 72$

put $A = 72$ in eq (2)

$$A = \frac{18}{r^2}$$

$$72 = \frac{18}{r^2}$$

$$72r^2 = 18$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{\cancel{18}^{1}}{\cancel{72}_{4}}$$

$$r^2 = \frac{1}{4}$$

Taking square root

$$\sqrt{r^2} = \sqrt{\frac{1}{4}}$$

$$\boxed{r = \frac{1}{2}}$$ Answer

Question #11 :- $a \propto \frac{1}{b^2}$ and $a=3$ when $b=4$, find $a$ when $b=8$

Sol

$$a \propto \frac{1}{b^2}$$

$$a = K \frac{1}{b^2}$$

$$a = \frac{K}{b^2} \longrightarrow (1)$$

put $a=3$ and $b=4$ (Given)

$$3 = \frac{K}{(4)^2}$$

$$3 = \frac{K}{16}$$



$$3 \times 16 = K$$

$$48 = K$$

put the value of "K" in eq (1)

$$\boxed{a = \frac{48}{b^2}} \longrightarrow (2)$$

Now

$a = ?$ when $b=8$

$$a = \frac{48}{(8)^2}$$

$$a = \frac{48}{64}$$ Divided by "16".

$$\boxed{a = \frac{3}{4}}$$ Answer.

**Question #13:-** $m \propto \frac{1}{n^3}$ and $m=2$

when $n=4$, find $m$ when $n=6$

and $n$ when $m=432$.

**Sol//**

$$m \propto \frac{1}{n^3}$$

$$m = K\frac{1}{n^3}$$

$$m = \frac{K}{n^3} \longrightarrow (1)$$

put $m=2$ and $n=4$ (Given)

$$2 = \frac{K}{(4)^3}$$

$$2 = \frac{K}{64}$$



$$2 \times 64 = K$$

$$\boxed{128 = K}$$

put the value of "K" in eq (1)

$$\boxed{m = \frac{128}{n^3}} \longrightarrow (2)$$

Now $m = ?$ when $n=6$

put $n=6$ in eq (2)

$$m = \frac{128}{(6)^3}$$

$$m = \frac{128}{216}$$ Divided by "8"

$$\boxed{m = \frac{16}{27}}$$ Answer

Now $n = ?$ when $m = 432$.

put $m = 432$ in eq (2)

$$m = \frac{128}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$432\, n^3 = 128$$

$$n^3 = \frac{128}{432}$$ Divided by 16



$$n^3 = \frac{8}{27}$$

$$n^3 = \frac{2^3}{3^3}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

Taking cube root

$$(n^3)^{1/3} = \left[\left(\frac{2}{3}\right)^3\right]^{1/3}$$

$$\boxed{n = \frac{2}{3}}$$ Answer

# EXERCISE # 3.3

Q#1 (i, iv, vi) Q#2 (ii, iv, v, vi)

Q#3 (i, iv) Q#4 (ii, iii)

**Question#1:-** Find a third proportional to

(i) 6 , 12

Sol//

$$6 : 12 :: 12 : c$$

Product of extremes = Product of means

$$(6)(c) = (12)(12)$$

$$c = \frac{(\cancel{12}^{2})(12)}{(\cancel{6})}$$



$$\boxed{c = 24}$$ Answer.

**Q#1 (iv)**

$$(x-y)^2 , x^3-y^3$$

Sol//

$$(x-y)^2 : x^3-y^3 :: x^3-y^3 : c$$

Product of extremes = Product of means

$$(x-y)^2(c) = (x^3-y^3)(x^3-y^3)$$

$$c = \frac{(x^3-y^3)(x^3-y^3)}{(x-y)^2}$$

$$c = \frac{\cancel{(x-y)}(x^2+xy+y^2)\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}\cancel{(x-y)}}$$

$$\boxed{c = (x^2+xy+y^2)^2}$$ Answer.

Q#1 (vi) $\frac{p^2-q^2}{p^3+q^3}, \frac{p-q}{p^2-pq+q^2}$

Sol//

$$\frac{p^2-q^2}{p^3+q^3} : \frac{p-q}{p^2-pq+q^2} :: \frac{p-q}{p^2-pq+q^2} : c$$

Product of extremes = Product of means

$$\left(\frac{p^2-q^2}{p^3+q^3}\right)(c) = \left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)$$

$$c = \frac{\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)}{\left(\frac{p^2-q^2}{p^3+q^3}\right)}$$



$$c = \left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right) \div \left(\frac{p^2-q^2}{p^3+q^3}\right)$$

$$c = \frac{(p-q)\quad(p-q)}{(p^2-pq+q^2)(p^2-pq+q^2)} \times \frac{p^3+q^3}{p^2-q^2}$$

$$c = \frac{(p-q)\quad(\cancel{p-q})}{(p^2-pq+q^2)(\cancel{p^2-pq+q^2})} \times \frac{(\cancel{p+q})(\cancel{p^2-pq+q^2})}{(\cancel{p-q})(\cancel{p+q})}$$

$$\boxed{c = \frac{p-q}{p^2-pq+q^2}}$$

Answer

**Question #2:-** Find the fourth proportional to

(ii) $4x^4, 2x^3, 18x^5$

Sol//

$$4x^2 : 2x^2 :: 18x^5 : c$$

Product of Extremes = Product of means

$$(4x^2)(c) = (2x^2)(18x^5)$$

$$c = \frac{(2\cancel{x^2})(18x^5)}{4\cancel{x^2}}$$

$$c = \frac{36x^5}{4}$$



$$\boxed{c = 9x^5}$$ Answer

**Q #2 (iv):-**

$x^2-11x+24, (x-3), 5x^4-40x^3$

Sol//

$$x^2-11x+24 : (x-3) :: 5x^4-40x^3 : c$$

Product of Extremes = Product of means

$$(x^2-11x+24)(c) = (x-3)(5x^4-40x^3)$$

$$c = \frac{(x-3)(5x^4-40x^3)}{(x^2-11x+24)}$$

Solve by factorization

$+24x^2$, $-11$: $1 \cdot 24$, $2 \cdot 12$, $-3x$, $-8x$

$x^2-11x+24$
$= x^2-3x-8x+24$
$= x(x-3)-8(x-3)$
$= (x-3)(x-8)$

$$c = \frac{\cancel{(x-3)}5x^3\cancel{(x-8)}}{\cancel{(x-3)}\cancel{(x-8)}}$$

$$\boxed{c = 5x^3}$$ Answer

**Q#2(v):-** $p^3+q^3, \; p^2-q^2, \; p^2-pq+q^2$

**Sol**

$$p^3+q^3 : p^2-q^2 :: p^2-pq+q^2 : c$$

Product of extremes = Product of means

$$(p^3+q^3)(c) = (p^2-q^2)(p^2-pq+q^2)$$

$$c = \frac{(p^2-q^2)(p^2-pq+q^2)}{(p^3+q^3)}$$

$$c = \frac{(p-q)\cancel{(p+q)}\cancel{(p^2-pq+q^2)}}{\cancel{(p+q)}\cancel{(p^2-pq+q^2)}}$$

$$\boxed{c = p-q}$$ Answer



**Q#2(vi):-**

$(p^2-q^2)(p^2+pq+q^2), \; p^3+q^3, \; p^3-q^3$

**Sol**

$$(p^2-q^2)(p^2+pq+q^2) : p^3+q^3 :: p^3-q^3 : c$$

Product of extremes = Product of means

$$(p^2-q^2)(p^2+pq+q^2)(c) = (p^3+q^3)(p^3-q^3)$$

$$c = \frac{(p^3+q^3)(p^3-q^3)}{(p^2-q^2)(p^2+pq+q^2)}$$

$$c = \frac{\cancel{(p+q)}(p^2-pq+q^2)\cancel{(p-q)}\cancel{(p^2+pq+q^2)}}{\cancel{(p-q)}\cancel{(p+q)}\cancel{(p^2+pq+q^2)}}$$

$$\boxed{c = (p^2-pq+q^2)}$$ Answer

**Question # 3 :-** Find the mean proportional between

(i) $20, 45$

Sol//

$$20 : m :: m : 45$$

Product of extremes = Product of means

$$(20)(45) = (m)(m)$$

$$900 = m^2$$

Taking Square root on both side

$$\sqrt{900} = \sqrt{m^2}$$

$\boxed{30 = \pm m}$ Answer.



**Question # 3 (iv) :-**

$$x^2 - y^2 , \frac{x-y}{x+y}$$

Sol//

$$x^2 - y^2 : m :: m : \frac{x-y}{x+y}$$

Product of Extremes = Product of means

$$(x^2-y^2)\left(\frac{x-y}{x+y}\right) = (m)(m)$$

$$(x-y)\cancel{(x+y)}\left(\frac{x-y}{\cancel{x+y}}\right) = m^2$$

$$(x-y)(x-y) = m^2$$

$$(x-y)^2 = m^2$$

Taking Square Root on both sides

$$\sqrt{(x-y)^2} = \sqrt{m^2}$$

$$\boxed{(x-y) = m}$$ Answer.

**Q#4(ii):- Find the value of the letter involved in the following continued proportions.**

$8 \; , \; x \; , \; 18$

Sol//

$8 : x :: x : 18$

Product of Extremes = Product of means



$(8)(18) = (x)(x)$

$144 = x^2$

Taking Square Root on both sides

$\sqrt{144} = \sqrt{x^2}$

$$\boxed{12 = x}$$ Answer.

**Q#4(iii):-** $12 \; , \; 3p-6 \; , \; 27$

Sol//

$12 : 3p-6 :: 3p-6 : 27$

Product of Extremes = Product of means

$$(12)(27) = (3p-6)(3p-6)$$

$$324 = (3p-6)^2$$

Taking square Root on both sides.

$$\sqrt{324} = \sqrt{(3p-6)^2}$$

$$\pm 18 = 3p-6$$

$$+18 = 3p-6$$



$$18+6 = 3p$$

$$24 = 3p$$

$$\frac{24}{3} = p$$

$$\boxed{8 = p}$$

$$-18 = 3p-6$$

$$-18+6 = 3p$$

$$-12 = 3p$$

$$\frac{-12}{3} = p$$

$$\boxed{-4 = p}$$

Answer

# EXERCISE # 3.4

Q#1(i, v, viii) , Q#2(ii, iv, v, vii)

**Question #1** Prove that

$a:b = c:d$ if

$$\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$$

Sol//

$$\frac{(4a+5b)}{(4a-5b)} = \frac{(4c+5d)}{(4c-5d)}$$

By Componendo - Dividendo Theorem

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)} = \frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$



$$\frac{4a+\cancel{5b}+4a-\cancel{5b}}{\cancel{4a}+5b-\cancel{4a}+5b} = \frac{4c+\cancel{5d}+4c-\cancel{5d}}{\cancel{4c}+5d-\cancel{4c}+5d}$$

$$\frac{\cancel{8}a}{\cancel{10}b} = \frac{\cancel{8}c}{\cancel{10}d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

Hence Proved

Q#1 (v)

$pa+qb : pa-qb = pc+qd : pc-qd$

Sol

$pa+qb : pa-qb = pc+qd : pc-qd$

$$\frac{(pa+qb)}{(pa-qb)} = \frac{(pc+qd)}{(pc-qd)}$$

Now Apply Componendo - Dividendo Theorem.

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)} = \frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+\cancel{qb}+pa-\cancel{qb}}{\cancel{pa}+qb-\cancel{pa}+qb} = \frac{pc+\cancel{qd}+pc-\cancel{qd}}{\cancel{pc}+qd-\cancel{pc}+qd}$$



$$\frac{\cancel{2p}a}{\cancel{2q}b} = \frac{\cancel{2p}c}{\cancel{2q}d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$a : b = c : d$

Hence proved

Q#I (viii) $\dfrac{a^2+b^2}{a^2-b^2} = \dfrac{ac+bd}{ac-bd}$

Sol

$$\frac{(a^2+b^2)}{(a^2-b^2)} = \frac{(ac+bd)}{(ac-bd)}$$

Now apply Componendo- Dividendo Theorem

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)} = \frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+\cancel{b^2}+a^2-\cancel{b^2}}{\cancel{a^2}+b^2-\cancel{a^2}+b^2} = \frac{ac+\cancel{bd}+ac-\cancel{bd}}{\cancel{ac}+bd-\cancel{ac}+bd}$$



$$\frac{\cancel{2}a^2}{\cancel{2}b^2} = \frac{\cancel{2}ac}{\cancel{2}bd}$$

$$\frac{\cancel{a}.a}{\cancel{b}.b} = \frac{\cancel{a}c}{\cancel{b}d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

Hence proved

Question # 2(ii) Using Theorem of Componendo - dividendo. Find the value of

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p},$$

if

$$m = \frac{10np}{n+p}$$

Sol

First we find the value of $\frac{m+5n}{m-5n}$

Given $$m = \frac{10np}{n+p}$$



$$m = \frac{(5n)(2p)}{n+p}$$

$$\frac{m}{5n} = \frac{2p}{n+p}$$

Apply Componendo - Dividendo Theorem

$$\frac{(m)+(5n)}{(m)-(5n)} = \frac{(2p)+(n+p)}{(2p)-(n+p)}$$

$$\frac{m+5n}{m-5n} = \frac{2p+n+p}{2p-n-p}$$

$$\boxed{\frac{m+5n}{m-5n} = \frac{3p+n}{p-n}} \longrightarrow (1)$$

Now we find the value of $\frac{m+5p}{m-5p}$

Given

$$m = \frac{10np}{n+p}$$

$$m = \frac{(5p)2n}{n+p}$$

$$\frac{m}{5p} = \frac{2n}{n+p}$$

Apply Componendo - Dividendo Theorem

$$\frac{(m)+(5p)}{(m)-(5p)} = \frac{(2n)+(n+p)}{(2n)-(n+p)}$$

$$\frac{m+5p}{m-5p} = \frac{2n+n+p}{2n-n-p}$$



$$\boxed{\frac{m+5p}{m-5p} = \frac{3n+p}{n-p}} \longrightarrow 2$$

Add eq 1 & 2

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = \frac{3p+n}{p-n} + \frac{3n+p}{n-p}$$

$$= \frac{3p+n}{p-n} + \frac{3n+p}{-(p-n)}$$

$$= \frac{3p+n}{p-n} - \frac{3n+p}{p-n}$$

$$= \frac{3p+n-3n-p}{p-n}$$ ★★★

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = \frac{2p-2n}{p-n}$$

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = \frac{2\cancel{(p-n)}}{\cancel{(p-n)}}$$

$$\boxed{\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = 2}$$ Answer.

**Question # 2 (iv) :-** Find the value of

$$\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} ,$$

if $x = \dfrac{3yz}{y-z}$

Sol

First we find the value of $\dfrac{x-3y}{x+3y}$

Given

$$x = \frac{3yz}{y-z}$$

$$\frac{x}{3y} = \frac{z}{y-z}$$

Apply Componendo - Dividendo Theorem.

$$\frac{(x)-(3y)}{(x)+(3y)} = \frac{(z)-(y-z)}{(z)+(y-z)}$$

$$\frac{x-3y}{x+3y} = \frac{z-y+z}{\cancel{z}+y-\cancel{z}}$$

$$\boxed{\frac{x-3y}{x+3y} = \frac{2z-y}{y}} \longrightarrow (1)$$

Now we find the value of $\frac{x+3z}{x-3z}$

Given

$$x = \frac{3yz}{y-z}$$

$$\frac{x}{3z} = \frac{y}{y-z}$$

Apply Componendo - Dividendo Theorem

$$\frac{(x)+(3z)}{(x)-(3z)} = \frac{(y)+(y-z)}{(y)-(y-z)}$$



$$\frac{x+3z}{x-3z} = \frac{y+y-z}{\cancel{y}-\cancel{y}+z}$$

$$\boxed{\frac{x+3z}{x-3z} = \frac{2y-z}{z}} \longrightarrow (2)$$

Subtract eq (2) from eq (1)

$$\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} = \frac{2z-y}{y} - \frac{2y-z}{z}$$

$$\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} = \frac{z(2z-y)-y(2y-z)}{yz}$$

$$\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} = \frac{2z^2-\cancel{zy}-2y^2+\cancel{yz}}{yz}$$

$$\boxed{\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} = \frac{2(z^2-y^2)}{yz}}$$ Answer

**Question # V:-** Using theorem of componendo-dividendo.

Find the value of

$$\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} , \quad \text{if } s = \frac{6pq}{p-q}$$

Sol

Find the value $\frac{s-3p}{s+3p}$ (first)

Given

$$s = \frac{6pq}{p-q}$$

$$s = \frac{(3p)(2q)}{p-q}$$



$$\frac{s}{3p} = \frac{2q}{p-q}$$

Using Componendo-dividendo Theorem

$$\frac{(s)-(3p)}{(s)+(3p)} = \frac{(2q)-(p-q)}{(2q)+(p-q)}$$

$$\frac{s-3p}{s+3p} = \frac{2q-p+q}{2q+p-q}$$

$$\boxed{\frac{s-3p}{s+3p} = \frac{3q-p}{q+p}} \longrightarrow (1)$$

Now find the value of $\frac{s+3q}{s-3q}$

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Given

$$s = \frac{6pq}{p-q}$$

$$s = \frac{(3q)(2p)}{p-q}$$

$$\frac{s}{3q} = \frac{2p}{p-q}$$

Using Componendo dividendo Theorem

$$\frac{(s)+(3q)}{(s)-(3q)} = \frac{(2p)+(p-q)}{(2p)-(p-q)}$$

$$\frac{s+3q}{s-3q} = \frac{2p+p-q}{2p-p+q}$$



$$\boxed{\frac{s+3q}{s-3q} = \frac{3p-q}{p+q}} \longrightarrow (2)$$

Add eq (1) and (2)

$$\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} = \frac{3q-p}{q+p} + \frac{3p-q}{p+q}$$

$$\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} = \frac{3q-p+3p-q}{q+p}$$

$$\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} = \frac{2q+2p}{q+p}$$

$$\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} = \frac{2(q+p)}{(q+p)}$$

$$\boxed{\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} = 2}$$

**Question # vii:-** Solve

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$$

Sol//

$$\frac{(\sqrt{x^2+2}+\sqrt{x^2-2})}{(\sqrt{x^2+2}-\sqrt{x^2-2})}=\frac{2}{1}$$

Apply Componendo dividendo Theorem.

$$\frac{(\sqrt{x^2+2}+\sqrt{x^2-2})+(\sqrt{x^2+2}-\sqrt{x^2-2})}{(\sqrt{x^2+2}+\sqrt{x^2-2})-(\sqrt{x^2+2}-\sqrt{x^2-2})}=\frac{(2)+(1)}{(2)-(1)}$$

$$\frac{\sqrt{x^2+2}+\cancel{\sqrt{x^2-2}}+\sqrt{x^2+2}-\cancel{\sqrt{x^2-2}}}{\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}-\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}}=\frac{3}{1}$$



$$\frac{\cancel{2}\sqrt{x^2+2}}{\cancel{2}\sqrt{x^2-2}}=\frac{3}{1}$$

Taking Square on both sides

$$\left(\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}}\right)^2=(3)^2$$

$$\frac{x^2+2}{x^2-2}=9$$

$$x^2+2=9(x^2-2)$$

$$x^2+2=9x^2-18$$

$$2+18=9x^2-x^2$$

$$20=8x^2$$

$$\frac{20}{8}=x^2$$

$$\frac{5}{2}=x^2$$

$$\sqrt{\frac{5}{2}}=\sqrt{x^2}$$

$$\boxed{\pm\sqrt{\frac{5}{2}}=x}$$ Answer

# EXERCISE # 3.5

## Question # 1 , 3 , 5

**Question #1:-** If $s$ varies directly as $u^2$ and inversely as $V$ and $s=7$ when $u=3$ , $v=2$. Find the value of $s$ when $u=6$ and $v=10$.

**Sol** According to Given Condition

$$S \propto \frac{u^2}{V}$$

$$S = K\frac{u^2}{V} \longrightarrow ①$$

$K = ?$ put $s=7$ , $u=3$ , $v=2$



$$7 = K\frac{(3)^2}{2}$$

$$7 = K\frac{9}{2}$$

$$\frac{7 \times 2}{9} = K$$

$$\boxed{\frac{14}{9} = K}$$

put the value of "K" in eq ①

$$\boxed{S = \frac{14}{9}\frac{u^2}{v}} \longrightarrow ②$$

Now

$S = ?$ when $u = 6$, $V = 10$

put these values in eq (2)

$$S = \frac{14}{9}\frac{u^2}{V}$$

$$S = \frac{14}{9}\frac{(6)^2}{10}$$

$$S = \frac{14}{\cancel{9}_1} \times \frac{\cancel{36}^4}{10}$$

$$S = 14 \times \frac{\cancel{4}^2}{\cancel{10}_5}$$

$$\boxed{S = \frac{28}{5}}$$ Answer.



Question#3:- If $y$ varies directly as $x^3$ and inversely as $z^2$ and $t$, and $y=16$ when $x=4$, $z=2$, $t=3$. Find the value of $y$ when $x=2$, $z=3$ and $t=4$.

Sol: According to the Given Condition

$$y \propto \frac{x^3}{z^2 t}$$

$$y = K\frac{x^3}{z^2 t} \longrightarrow (1)$$

put $y=16$, $x=4$, $z=2$, $t=3$ in eq ①

$$y = \frac{Kx^3}{z^2 t}$$

$$16 = \frac{K(4)^3}{(2)^2(3)}$$

$$16 = \frac{K\,\cancel{64}^{16}}{\cancel{(4)}_1(3)}$$

$$16 = \frac{K(16)}{3}$$

$$16 \times 3 = K(16)$$

$$3 = \frac{K\cancel{(16)}}{\cancel{16}}$$



$$\boxed{3 = K}$$

put the value of "K" in eq ①

$$\boxed{y = \frac{3x^3}{z^2 t}} \longrightarrow ②$$

Now $y = ?$ when $x=2$, $z=3$ and $t=4$

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$y = \frac{3\,\cancel{(8)}^2}{9\,\cancel{(4)}_1}$$

$$y = \frac{\cancel{6}^2}{\cancel{9}_3} \Rightarrow \boxed{y = \frac{2}{3}}$$ Answer.

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Question#5:- If $v$ varies directly as the product $xy^3$ and inversely as $z^2$ and $v=27$ when $x=7$, $y=6$, $z=7$. Find the value of $v$ when $x=6$, $y=2$, $z=3$.

Sol: According to Given Condition.

$$v \propto \frac{xy^3}{z^2}$$

$$v = K\frac{xy^3}{z^2} \longrightarrow ①$$

$K=?$

put $v=27$, $x=7$, $y=6$, $z=7$



$$27 = K\frac{(7)(6)^3}{(7)^2}$$

$$27 = K\frac{(\cancel{7})(216)}{\cancel{49}\,7}$$

$$\frac{27\times7}{216} = K \qquad \text{Divided by 3}$$

$$\frac{9\times7}{72} = K$$

$$\frac{\cancel{9}\times7}{\cancel{72}\,8} = K$$

$$\boxed{\frac{7}{8} = K}$$

put the value of K in eq 1

$$\boxed{V = \frac{7}{8}\frac{xy^3}{z^2}} \longrightarrow 2$$

Now

V=? when $x=6$, $y=2$, $z=3$ in eq 2

$$V = \frac{7}{8}\frac{(6)(2)^3}{(3)^2}$$

$$V = \frac{7}{\cancel{8}}\frac{(6)\cancel{(8)}}{(9)}$$

$$V = \frac{7\,\cancel{(6)}^{2}}{\cancel{9}\,3}$$



$$\boxed{V = \frac{14}{3}}$$ Answer

# EXERCISE # 3.6

Q#1(ii, iii, vi) , Q#2(ii)

**Question #1(ii):-** If $a:b = c:d$ $(a, b, c, d \neq 0)$, then show that

$$\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

Sol

Let $a:b = c:d$

$$\frac{a}{b} = \frac{c}{d} = K$$

$$\frac{a}{b} = K \quad ; \quad \frac{c}{d} = K$$

$$a = bK \quad ; \quad c = dK$$



$$R.H.S = \frac{6c-5d}{6c+5d}$$

put the value of "c"

$$= \frac{6dK-5d}{6dK+5d}$$

$$= \frac{\cancel{d}(6K-5)}{\cancel{d}(6K+5)}$$

$$= \frac{6K-5}{6K+5} \longrightarrow (1)$$

$$L.H.S = \frac{6a-5b}{6a+5b}$$

put the value of "a"

$$= \frac{6bK - 5b}{6bK + 5b}$$

$$= \frac{\cancel{b}(6K-5)}{\cancel{b}(6K+5)}$$

$$= \frac{6K-5}{6K+5} \longrightarrow (2)$$

from equation (1) & (2)

L.H.S = R.H.S

$$\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

Hence proved.

**Question #1 (iii)**



$$\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

Let $a : b = c : d$

$$\frac{a}{b} = \frac{c}{d} = K$$

$$\frac{a}{b} = K \quad ; \quad \frac{c}{d} = K$$

$$\boxed{a = bK} \quad ; \quad \boxed{c = dK}$$

$$R.H.S = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

put the values of a & c

$$= \sqrt{\frac{(bK)^2+(dK)^2}{b^2+d^2}}$$

$$= \sqrt{\frac{b^2K^2+d^2K^2}{b^2+d^2}}$$ $a=bK$ & $c=dK$

$$= \sqrt{\frac{K^2\cancel{(b^2+d^2)}}{\cancel{(b^2+d^2)}}}$$

$$= \sqrt{K^2}$$

$$= K \longrightarrow (1)$$

$$L.H.S = \frac{a}{b}$$

put the value of "a".



$a=bK$

$$= \frac{\cancel{b}K}{\cancel{b}}$$

$$= K \longrightarrow (2)$$

from equation (1) & (2)

L.H.S = R.H.S

$$\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

Hence proved

## Question # 1 (vi):-

$$a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$$

Sol//

$$a : b = c : d$$

let $\frac{a}{b} = \frac{c}{d} = K$

$$\frac{a}{b} = K \quad ; \quad \frac{c}{d} = K$$

$$\boxed{a = bK} \quad ; \quad \boxed{c = dK}$$

$$R.H.S = c^2+d^2 : \frac{c^3}{c+d}$$

$$= c^2+d^2 : \frac{c^3}{c+d}$$



put the value of "c".

$$= (dK)^2 + d^2 : \frac{(dK)^3}{dK+d}$$

$$= d^2K^2 + d^2 : \frac{d^3K^3}{d(K+1)}$$

$$= d^2(K^2+1) : \frac{dd\cancel{d}K^3}{\cancel{d}(K+1)}$$

$$= d^2(K^2+1) : \frac{d^2K^3}{(K+1)}$$

$$= \frac{\cancel{d^2}(K^2+1)}{\frac{\cancel{d^2}K^3}{(K+1)}}$$

$$= \boxed{\frac{(K^2+1)(K+1)}{K^3}} \longrightarrow (1)$$

$$L.H.S = a^2+b^2 : \frac{a^3}{a+b}$$

put the value of $a = bK$

$$= (bK)^2 + b^2 : \frac{(bK)^3}{bK+b}$$

$$= b^2K^2 + b^2 : \frac{b^3K^3}{b(K+1)}$$

$$= b^2(K^2+1) : \frac{bb\cancel{b}K^3}{\cancel{b}(K+1)}$$

$$= b^2(K^2+1) : \frac{b^2K^3}{(K+1)}$$



$$= \frac{\cancel{b^2}(K^2+1)}{\frac{\cancel{b^2}K^3}{(K+1)}}$$

$$= \boxed{\frac{(K^2+1)(K+1)}{K^3}} \longrightarrow 2$$

From eq 1 and 2

L.H.S = R.H.S

$$a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$$

Hence proved

**Question#2(ii)** :- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ $(a, b, c, d, e, f \neq 0)$ then show that

$$\frac{ac + ce + ea}{bd + df + fb} = \left[\frac{ace}{bdf}\right]^{2/3}$$

Sol//

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$

$\frac{a}{b} = K$, $\frac{c}{d} = K$, $\frac{e}{f} = K$

$\boxed{a = bK}$ $\boxed{c = dK}$ $\boxed{e = fK}$

$$R.H.S = \left[\frac{ace}{bdf}\right]^{2/3}$$



put the value of a, c, e

$$= \left[\frac{\cancel{b}K\, \cancel{d}K\, \cancel{f}K}{\cancel{b}\,\cancel{d}\,\cancel{f}}\right]^{2/3}$$

$$= [K\;K\;K]^{2/3}$$

$$= [K^{\cancel{3}}]^{2/\cancel{3}}$$

$$= K^2 \longrightarrow (1)$$

$$L.H.S = \frac{ac + ce + ea}{bd + df + fb}$$

put the value of a, c and e.

$$= \frac{bKdK + dKfK + fKbK}{bd + df + fb}$$

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$$= \frac{bdK^2 + dfK^2 + fbK^2}{bd + df + fb}$$

$$= \frac{K^2 \cancel{(bd + df + fb)}}{\cancel{(bd + df + fb)}}$$

$$= K^2 \longrightarrow (2)$$

From eq (1) and (2)

$$L.H.S = R.H.S$$

$$\frac{ac + ce + ea}{bd + df + fb} = \left[\frac{ace}{bdf}\right]^{2/3}$$

Hence proved

# EXERCISE # 3.7

## Question # 2 , 3 , 9

**Question#2:-** The surface area $S$ of the sphere varies directly as the square of radius $r$, and $S = 16\pi$ when $r = 2$. Find $r$ when $S = 36\pi$.

**Sol//** According to Given Condition

$$S \propto r^2$$

$$S = K r^2 \longrightarrow (1)$$

put $S = 16\pi$ and $r = 2$ (Given)

$$16\pi = K(2)^2$$



$$16\pi = K(4)$$

$$\frac{\cancel{16}^{\,4}\pi}{\cancel{4}_{\,1}} = K$$

$$\boxed{4\pi = K}$$

put the value of "K" in eq (1)

$$\boxed{S = 4\pi r^2} \longrightarrow (2)$$

Now $r = ?$ when $S = 36\pi$

put the value of "S" in eq (2)

$$36\pi = 4\pi r^2$$

$$\frac{{}^{9}\cancel{36}\cancel{\pi}}{\cancel{4}\cancel{\pi}} = r^2$$

$$9 = r^2$$

Taking square root

$$\sqrt{9} = \sqrt{r^2}$$

$$\boxed{3 = r}$$ Answer.

Question #3:- In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and F = 32 lb when S = 1.6 in.

Find (i) S when F = 50 lb

(ii) F when S = 0.8 in

Sol: According to Given Condition.

$$F \propto S$$

$$F = KS \longrightarrow (1)$$

put the value of F and S

$$32 = K(1.6)$$



$$\frac{32}{1.6} = K$$

$$\boxed{20 = K}$$

put the value of "K" in eq (1)

$$\boxed{F = 20S} \longrightarrow (2)$$

Now $S = ?$ when $F = 50\,lb$

put the value of "F" in eq (2)

$$50 = 20S$$

$$\frac{50}{20} = S$$

$$\boxed{2.5 \text{ in} = S}$$ Answer.

Now $F = ?$ when $S = 0.8$ in

Put the value of "S" in eq (2)

$$F = 20S$$

$$F = 20(0.8)$$

$$\boxed{F = 16\,lb}$$ Answer

Question#9:- The kinetic energy (K.E) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the K.E is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec, determine the K.E of a 3000 lb automobile travelling 44 ft/sec.



Sol// According to Given Condition.

$$K.E \propto mv^2$$

$$K.E = Kmv^2 \longrightarrow (1)$$

put K.E = 4320 ft/lb, m = 45 lb, v = 24 ft/sec in eq (1)

$$4320 = K(45)(24)^2$$

$$4320 = K(45)(576)$$

$$\frac{4320}{(45)(576)} = K$$

$$\frac{4320}{25920} = K$$

$$\boxed{\frac{1}{6} = K}$$

put the value of "K" in eq 1

$$K.E = K m v^2$$

$$\boxed{K.E = \frac{1}{6} m v^2} \longrightarrow 2$$

$K.E = ?$ when $m = 3000\ lb$

and $v = 44\ ft/sec$

put these values in eq 2

$$K.E = \frac{1}{6}(3000)(44)^2$$

$$K.E = \frac{1}{\cancel{6}_{1}}(\cancelto{500}{3000})(1936)$$



$$K.E = (500)(1936)$$

$$\boxed{K.E = 968000\ ft/lb}$$

Answer

# MISCELLANEOUS EXERCISE #3

**Q#1 :-** MCQs (Doyourself from Book)

**Q#2:-** Write short answers of the following Questions.

**Q#2 (i):-** Define ratio and give one example.

**Ans:-** A relation between two quantities of the same kind is called ratio.

Example:-

Let a, b be any two quantities of the same kind, So their ratio is written as $a : b$.



**Q#2 (ii)** Define Proportion.

**Ans:-** A proportion is a statement, which is expressed as an equivalence of two ratio as

Example:- $a : b :: c : d$

**Q#2 (iii)** Define Direct variation.

**Ans:-** If two quantities are related in such a way that increase (decrease) in one quantity cause increase (decrease) in the other quantity.

**Q#2 (iv)** Define inverse variation.

**Ans:-** If two quantities are related in such a way that when quantity increases, the other decrease is called inverse variation.

Q#2(v):- State theorem of componendo-dividendo.

Ans:- If $\frac{a}{b} = \frac{c}{d}$

then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Q#2(vi):- Find $x$, if $6:x::3:5$

Sol

$$6:x::3:5$$

Product of extremes = Product of means

$$(6)(5) = (x)(3)$$



$$30 = 3x$$

$$\frac{30}{3} = x$$

$$\boxed{10 = x}$$ Answer

Q#2(vii) If $x$ and $y^2$ varies directly, and $x=27$ when $y=4$. Find the value of $y$ when $x=3$

Sol

$$x \propto y^2$$

$$x = ky^2 \longrightarrow (1)$$

$K = ?$ put $x = 27$ and $y = 4$ in eq (1)

$$x = K y^2$$

$$27 = K(4)^2$$

$$27 = K(16)$$

$$\boxed{\frac{27}{16} = K}$$

put the value of "K" in eq (1)

$$x = K y^2$$

$$\boxed{x = \frac{27}{16} y^2} \longrightarrow (2)$$

Now

$y = ?$ when $x = 3$



put $x = 3$ in eq (3)

$$3 = \frac{27}{16} y^2$$

$$\frac{\cancel{3}^{1} \times 16}{\cancel{27}_{9}} = y^2$$

$$\frac{16}{9} = y^2$$

Taking square root on both side

$$\sqrt{\frac{16}{9}} = \sqrt{y^2}$$

$$\boxed{\pm \frac{4}{3} = y}$$ Answer sir.

Q#2(viii) If u and v varies inversely, and u=8, when v=3. Find v when u=12

Sol

$$u \propto \frac{1}{v}$$

$$u = k\frac{1}{v} \longrightarrow (1)$$

put u=8 and v=3 in eq (1)

$$8 = k\left(\frac{1}{3}\right)$$

$$8 \times 3 = k$$

$$\boxed{24 = k}$$

put the value of "k" in eq (1)



$$u = 24\left(\frac{1}{v}\right)$$

$$\boxed{u = \frac{24}{v}} \longrightarrow (2)$$

Now

v=? when u=12

put u=12 in eq (2)

$$12 = \frac{24}{v}$$

$$12v = 24$$

$$v = \frac{24}{12}$$

$$\boxed{v = 2}$$ Answer

Q#2 (ix):- Find the fourth proportional 8, 7, 6

Sol// let "x" be the $4^{th}$ proportional.

$$8 : 7 :: 6 : x$$

$$(8)(x) = (7)(6)$$

$$8x = 42$$

$$x = \frac{42}{8}$$ Divided by 2

$$\boxed{x = \frac{21}{4}}$$ Answer



Q#2 (x):- Find a mean proportional to 16 to 49.

Sol// let x be the mean proportional

$$16 : x :: x : 49$$

$$(16)(49) = x^2$$

$$784 = x^2$$

Taking square root on both sides

$$\sqrt{784} = \sqrt{x^2}$$

$$\boxed{28 = x}$$ Answer

Q#2(xi):- Find a third proportional to 28 and 4.

Sol//
let "$x$" be the third proportional.

$$28 : 4 :: 4 : x$$

$$(28)(x) = (4)(4)$$

$$28x = 16$$

$$x = \frac{16}{28}$$

Divided by 4

$$\boxed{x = \frac{4}{7}}$$

Answer sir.



Q#2(xii) If $y \propto \frac{x^2}{z}$ and $y = 28$ when $x = 7$, $z = 2$, then find $y$.

Sol//

$$y \propto \frac{x^2}{z}$$

$$y = K\frac{x^2}{z} \longrightarrow ①$$

K=? put $y = 28$, $x = 8$ and $z = 2$ in eq①

$$28 = \frac{K(7)^2}{2}$$

$$28 = \frac{K(49)}{2}$$

$$28 \times 2 = K(49)$$

$$\frac{56}{49} = K$$

Divided by 7

$$\boxed{\frac{8}{7} = K}$$

put the value of K in eq (1)

$$y = K\frac{x^2}{z}$$

$$\boxed{y = \frac{8}{7}\frac{x^2}{z}} \longrightarrow (2)$$

$y = ?$ put $x = 7$ and $z = 2$



$$y = \frac{8}{7}\frac{(7)^2}{2}$$

$$y = \frac{\cancel{8}^{4}}{\cancel{7}_{1}}\left(\frac{\cancel{49}^{7}}{\cancel{2}_{1}}\right)$$

$$y = 4(7)$$

$$\boxed{y = 28}$$ Answer.

Q#2(xiii) If $z \propto xy$ and $z = 36$ when $x = 2$, $y = 3$, then find $z$.

Sol/

$$z \propto xy$$

$$z = Kxy \longrightarrow (1)$$

$K = ?$ when $Z = 36$, $x = 2$ and $y = 3$

$$36 = K(2)(3)$$

$$36 = K(6)$$

$$\frac{36}{6} = K$$

$$\boxed{6 = K}$$

put the value of "K" in eq ①

$$\boxed{Z = 6xy} \longrightarrow ②$$

Now

$Z = ?$ when $x = 2$ and $y = 3$

② $\Rightarrow$ $Z = 6xy$



$$Z = 6(2)(3)$$

$$Z = 6(6)$$

$$\boxed{Z = 36}$$ Answer

Q#2(xiv) If $w \propto \frac{1}{v^2}$ and $w = 2$ when $v = 3$, then find $w$.

Sol

$$w \propto \frac{1}{v^2}$$

$$W = K\frac{1}{v^2}$$

$$W = \frac{K}{v^2} \longrightarrow ①$$

$K = ?$ when $W = 2$ and $V = 3$

$$W = \frac{K}{V^2}$$

$$2 = \frac{K}{(3)^2}$$

$$2 = \frac{K}{9}$$

$$2 \times 9 = K$$

$$\boxed{18 = K}$$

put the value of "K" in eq 1

$$W = \frac{K}{V^2}$$

$$\boxed{W = \frac{18}{V^2}} \rightarrow 2$$



Now

$W = ?$ when $V = 3$

$$2 \Rightarrow \quad W = \frac{18}{(3)^2}$$

$$W = \frac{18}{9}$$

$$\boxed{W = 2}$$ Answer.

# Unit # 4

## Partial Fractions

### EXERCISE # 4.1 Question# 2, 4, 7, 8

**Question # 2 :-** Resolve into partial fractions.

$$\frac{x-11}{(x-4)(x+3)}$$

Sol//

$$\frac{x-11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)} \longrightarrow (1)$$

Multiply by $(x-4)(x+3)$ on both sides

$$\frac{x-11}{\cancel{(x-4)}\cancel{(x+3)}}\cancel{(x-4)}\cancel{(x+3)} = \frac{A}{\cancel{(x-4)}}\cancel{(x-4)}(x+3) + \frac{B}{\cancel{(x+3)}}(x-4)\cancel{(x+3)}$$



$$x-11 = A(x+3) + B(x-4) \longrightarrow (2)$$

put $x-4=0$

$x=4$ in eq (2)

$$4-11 = A(4+3) + B(4-4)$$

$$-7 = A(7) + B(0)$$

$$-7 = 7A + 0$$

$$-7 = 7A$$

$$\frac{-7}{7} = A$$

$$\boxed{-1 = A}$$

Now put $x+3=0$

$x=-3$ in eq ②

$x-11 = A(x+3)+B(x-4)$

$-3-11 = A(-3+3)+B(-3-4)$

$-14 = A(0)+B(-7)$

$-14 = 0-7B$

$\cancel{-}14 = \cancel{-}7B$

$\frac{14}{7} = B$

$\boxed{2 = B}$

put the value of "A" and "B" in eq ①



$$\boxed{\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}}$$

Answer.

**Question # 4:- Resolve into partial fractions.**

$$\frac{x-5}{x^2+2x-3}$$

Sol// First we factorize the

$x^2+2x-3$

$= x^2-1x+3x-3$

$= x(x-1)+3(x-1)$

$= (x-1)(x+3)$

$-3x^2$

$-1x \quad +3x$

$$\frac{x-5}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)} \longrightarrow (1)$$

Multiply $(x-1)(x+3)$ on both sides

$$\frac{x-5}{\cancel{(x-1)}\cancel{(x+3)}} \cancel{(x-1)}\cancel{(x+3)} = \frac{A}{\cancel{(x-1)}} \cancel{(x-1)}(x+3) + \frac{B}{\cancel{(x+3)}} (x-1)\cancel{(x+3)}$$

$$x-5 = A(x+3) + B(x-1) \longrightarrow (2)$$

put $x-1=0$

$x=1$ in eq (2)

$$1-5 = A(1+3) + B(1-1)$$



$$-4 = A(4) + B(0)$$

$$-4 = 4A + 0$$

$$-4 = 4A$$

$$-\frac{4}{4} = A$$

$$\boxed{-1 = A}$$

Now

put $x+3=0$

$x=-3$ in eq (2)

$$x-5 = A(x+3) + B(x-1)$$

$$-3-5 = A(-3+3) + B(-3-1)$$

$$-8 = A(0) + B(-4)$$

$$-8 = 0 - 4B$$

$\cancel{+}8 = \cancel{+}4B$

$\frac{8}{4} = B$

$\boxed{2 = B}$

put the value of "A" and "B" in eq ①

$$\frac{x-5}{(x-1)(x+3)} = \frac{-1}{(x-1)} + \frac{2}{(x+3)}$$

OR

$$\boxed{\frac{x-5}{x^2+2x-3} = \frac{-1}{(x-1)} + \frac{2}{(x+3)}}$$

Answer



Q#7:- Resolve into partial fractions.

$$\frac{x^2+2x+1}{(x-2)(x+3)}$$

Sol

first we solve $(x-2)(x+3)$

$$(x-2)(x+3) = x^2+3x-2x-6$$

$$= x^2+x-6$$

$$\frac{x^2+2x+1}{(x-2)(x+3)} = \frac{x^2+2x+1}{x^2+x-6}$$

improper fraction

Now we change into proper fraction

first

| | 1 |
|---|---|
| $x^2+x-6$ | $\cancel{x^2}+2x+1$ |
| | $\cancel{x^2}+x-6$ |
| | $-\quad-\quad+$ |
| | $x+7$ |

third

2nd

$\frac{\cancel{x^2}}{\cancel{x^2}}$ 1

$$\frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{(x+7)}{x^2+x-6} \longrightarrow (1)$$

Only Solve $\frac{x+7}{x^2+x-6} = \frac{x-7}{(x-2)(x+3)}$



$$\frac{x-7}{(x-2)(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x+3)} \longrightarrow (2)$$

Multiply $(x-2)(x+3)$ on both sides

$$\frac{x-7}{\cancel{(x-2)}\cancel{(x+3)}}\cancel{(x-2)}\cancel{(x+3)} = \frac{A}{\cancel{(x-2)}}\cancel{(x-2)}(x+3) + \frac{B}{\cancel{(x+3)}}(x-2)\cancel{(x+3)}$$

$$x-7 = A(x+3) + B(x-2) \longrightarrow (3)$$

Put $x-2=0$

$x=2$ in eq (3)

$$2-7 = A(2+3) + B(2-2)$$

$$-5 = A(5) + B(0)$$

$$-5 = 5A + 0$$

$$-5 = 5A$$

$$\frac{-5}{5} = A$$

$$\boxed{-1 = A}$$

put

$$x + 3 = 0$$

$$x = -3 \quad \text{in eq } 3$$

$$x - 7 = A(x+3) + B(x-2)$$

$$-3 - 7 = A(-3+3) + B(-3-2)$$



$$-10 = A(0) + B(-5)$$

$$-10 = 0 - 5B$$

$$\cancel{-}10 = \cancel{-}5B$$

$$\frac{10}{5} = B$$

$$\boxed{2 = B}$$

Put the value of "A" and "B" in eq 2

$$\frac{x-7}{(x-2)(x+3)} = \frac{-1}{(x-2)} + \frac{2}{(x+3)}$$

Now put the value of $\frac{x-7}{(x-2)(x+3)}$ in eq 1

$$\frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{-1}{(x-2)} + \frac{2}{(x+3)}$$

OR

$$\boxed{\frac{x^2+2x+1}{(x-2)(x+3)} = 1 - \frac{1}{(x-2)} + \frac{2}{(x+3)}}$$ Answer.

<u>Question #8:-</u> Resolve into partial fractions.

$$\frac{6x^3+5x^2-7}{3x^2-2x-1}$$

Sol//

$\frac{6x^3+5x^2-7}{3x^2-2x-1} \Rightarrow$ improper fraction



first we change into proper fraction.

$$\begin{array}{r|l} & 2x+3 \\ \hline 3x^2-2x-1 & 6x^3+5x^2-7 \\ & -6x^3 \mp 4x^2 \mp 2x \\ \hline & 9x^2+2x-7 \\ & -9x^2 \mp 6x \mp 3 \\ \hline & 8x-4 \end{array}$$

$\frac{\cancel{6}^{2}x^3}{\cancel{3}x^2} = 2x$

$\frac{\cancel{9}^{3}x^2}{\cancel{3}x^2} = 3$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3) + \frac{(8x-4)}{(3x^2-2x-1)} \longrightarrow (1)$$

Now first we Solve

$$\frac{8x-4}{3x^2-2x-1}$$

Solve by factorization

$3x^2-2x-1$

$= 3x^2+1x-3x-1$

$= x(3x+1)-1(3x+1)$

$= (3x+1)(x-1)$

$3x^2$: $+1x$, $-3x$

Replace $3x^2-2x-1$ by $(3x+1)(x-1)$

$$\frac{8x-4}{(3x+1)(x-1)} = \frac{A}{(3x+1)} + \frac{B}{(x-1)} \longrightarrow 2$$



Multiply by $(3x+1)(x-1)$ on both sides.

$$\frac{8x-4}{\cancel{(3x+1)}\cancel{(x-1)}}\cancel{(3x+1)}\cancel{(x-1)} = \frac{A}{\cancel{(3x+1)}}\cancel{(3x+1)}(x-1) + \frac{B}{\cancel{(x-1)}}(3x+1)\cancel{(x-1)}$$

$$8x-4 = A(x-1) + B(3x+1) \longrightarrow 3$$

put

$3x+1=0$

$3x=-1$

$x = \frac{-1}{3}$ in eq 3

$$8\left(\frac{-1}{3}\right)-4 = A\left(\frac{-1}{3}-1\right) + B\left(\cancel{3}\left(\frac{-1}{\cancel{3}}\right)+1\right)$$

$$-\frac{8}{3}-4 = A\left(\frac{-1-3}{3}\right) + B(-1+1)$$

$$\frac{-8-12}{3} = A\left(-\frac{4}{3}\right) + B(0)$$

$$-\frac{20}{3} = -\frac{4}{3}A + 0$$

$$\cancel{-}\frac{20}{3} = \cancel{-}\frac{4}{3}A$$

$$\frac{\cancel{20}^{5}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}_{1}} = A$$

$$\boxed{5 = A}$$

Now

put $x-1=0$

$x=1$ in eq (3)

$$8x-4 = A(x-1) + B(3x+1)$$



$$8(1)-4 = A(1-1) + B(3(1)+1)$$

$$8-4 = A(0) + B(3+1)$$

$$4 = 0 + B(4)$$

$$4 = 4B$$

$$\frac{4}{4} = B$$

$$\boxed{1 = B}$$

Put the value of "A" and "B" in eq (2)

$$\frac{8x-4}{(3x+1)(x-1)} = \frac{5}{(3x+1)} + \frac{1}{(x-1)}$$

Now put the value of

$$\frac{8x-4}{(3x+1)(x-1)}$$

OR

$$= \frac{8x-4}{3x^2-2x-1} \quad \text{in eq } 1$$

$$\boxed{\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3) + \frac{5}{(3x+1)} + \frac{1}{(x-1)}}$$

Answer.

# EXERCISE # 4.2

## Question # 1 , 2 , 6 , 8

**Question #1:-** Resolve into partial fractions.

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}$$

Sol/

$$\frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2} + \frac{C}{(x+3)} \longrightarrow (1)$$

Multiply $(x+2)^2(x+3)$ on both sides.

$$\frac{x^2+7x+11}{\cancel{(x+2)^2}\cancel{(x+3)}}\cancel{(x+2)^2}\cancel{(x+3)} = \frac{A}{\cancel{(x+2)}}(x+2)^{\cancel{2}}(x+3) + \frac{B}{\cancel{(x+2)^2}}\cancel{(x+2)^2}(x+3)$$

$$+ \frac{C}{\cancel{(x+3)}}(x+2)^2\cancel{(x+3)}$$



$$x^2+7x+11 = A(x+2)(x+3) + B(x+3) + C(x+2)^2 \longrightarrow (2)$$

$$x^2+7x+11 = A(x^2+3x+2x+6) + B(x+3) + C((x)^2+(2)^2+2(x)(2))$$

$$x^2+7x+11 = A(x^2+5x+6) + B(x+3) + C(x^2+4+4x)$$

$$x^2+7x+11 = Ax^2+5Ax+6A+Bx+3B+Cx^2+4C+4Cx$$

$$x^2+7x+11 = Ax^2+Cx^2+5Ax+Bx+4Cx+6A+3B+4C$$

$$1x^2+7x+11 = (A+C)x^2+(5A+B+4C)x+(6A+3B+4C)$$

Comparing Coefficient of $x^2$, $x$ and Constant

$1 = A+C \longrightarrow (3)$

$7 = 5A+B+4C \longrightarrow (4)$

$11 = 6A+3B+4C \longrightarrow (5)$

put $x+2=0$

$x=-2$ in eq (2)

$$x^2+7x+11 = A(x+2)(x+3) + B(x+3) + C(x+2)^2$$

$$(-2)^2+7(-2)+11 = A\underbrace{\cancel{(-2+2)}}_{=0}(-2+3) + B(-2+3) + C\underbrace{\cancel{(-2+2)}^2}_{=0}$$

$$4-14+11 = B(1)$$

$$15-14 = B$$

$$\boxed{1 = B}$$

put $x+3=0$

$x=-3$ in eq (2)

$$x^2+7x+11 = A(x+2)(x+3) + B(x+3) + C(x+2)^2$$



$$(-3)^2+7(-3)+11 = A(-3+2)\underbrace{\cancel{(-3+3)}}_{=0} + B\underbrace{\cancel{(-3+3)}}_{=0} + C(-3+2)^2$$

$$9-21+11 = C(-1)^2$$

$$+20-21 = C(1)$$

$$\boxed{-1 = C}$$

put the value of "C" in eq (3)

$$1 = A+C \qquad ; C=-1$$

$$1 = A+(-1)$$

$$1 = A-1$$

$$1+1 = A$$

$$\boxed{2 = A}$$

put the value of A, B and C in equation # 1.

$$\frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{2}{(x+2)} + \frac{1}{(x+2)^2} + \frac{-1}{(x+3)}$$

$$\boxed{\frac{x^2+7x+11}{(x+2)^2(x+3)} = \frac{2}{(x+2)} + \frac{1}{(x+2)^2} - \frac{1}{(x+3)}}$$ Answer.

## Question # 2 :-

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}$$

Sol

$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)} \longrightarrow ①$$

Multiply $(x-1)^2(x-2)$ on both sides.



$$\frac{x^2-3x+1}{\cancel{(x-1)^2}\cancel{(x-2)}}\cancel{(x-1)^2}\cancel{(x-2)} = \frac{A}{\cancel{(x-1)}}(x-1)^2(x-2) + \frac{B}{\cancel{(x-1)^2}}\cancel{(x-1)^2}(x-2)$$
$$+ \frac{C}{\cancel{(x-2)}}(x-1)^2\cancel{(x-2)}$$

$$x^2-3x+1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2 \longrightarrow ②$$

$$x^2-3x+1 = A(x^2-2x-1x+2) + B(x-2) + C\left((x)^2+(1)^2-2(x)(1)\right)$$

$$x^2-3x+1 = A(x^2-3x+2) + B(x-2) + C(x^2+1-2x)$$

$$x^2-3x+1 = Ax^2-3Ax+2A+Bx-2B+Cx^2+C-2Cx$$

$$x^2-3x+1 = Ax^2+Cx^2+Bx-3Ax-2Cx+2A-2B+C$$

$$1x^2-3x+1 = (A+C)x^2+(B-3A-2C)x+(2A-2B+C)$$

Comparing Coefficient of $x^2$, $x$ & constant.

$$1 = A + C \longrightarrow (3)$$

$$-3 = B - 3A - 2C \longrightarrow (4)$$

$$1 = 2A - 2B + C \longrightarrow (5)$$

put $x - 1 = 0$

$x = 1$ in equation # 2

$$x^2 - 3x + 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$(1)^2 - 3(1) + 1 = A\underbrace{\cancel{(1-1)}}_{=0}(1-2) + B(1-2) + C\underbrace{\cancel{(1-1)}^2}_{=0}$$

$$1 - 3 + 1 = B(-1)$$

$$2 - 3 = B(-1)$$



$$\cancel{-}1 = \cancel{-}B$$

$$\boxed{1 = B}$$

put $x - 2 = 0$

$x = 2$ in equation # 2

$$x^2 - 3x + 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$(2)^2 - 3(2) + 1 = A(2-1)\underbrace{\cancel{(2-2)}}_{=0} + B\underbrace{\cancel{(2-2)}}_{=0} + C(2-1)^2$$

$$4 - 6 + 1 = C(1)^2$$

$$5 - 6 = C(1)$$

$$\boxed{-1 = C}$$

put the value of "C" in eq (3)

$$1 = A + C$$

$$1 = A + (-1)$$

$$1 = A - 1$$

$$1 + 1 = A$$

$$\boxed{2 = A}$$

put the value of "A", "B" and "C" in equation # 1.

$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{(x-1)} + \frac{1}{(x-1)^2} + \frac{-1}{(x-2)}$$



$$\boxed{\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{(x-1)} + \frac{1}{(x-1)^2} - \frac{1}{x-2}}$$

Answer

## Question # 6 :- Resolve into partial fractions.

$$\frac{1}{(x-1)^2(x+1)}$$

Sol/

$$\frac{1}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)} \longrightarrow (1)$$

Multiply $(x-1)^2(x+1)$ on both sides.

$$\frac{1}{\cancel{(x-1)^2(x+1)}}\cancel{(x-1)^2(x+1)} = \frac{A}{\cancel{(x-1)}}(x-1)^{\cancel{2}}(x+1) + \frac{B}{\cancel{(x-1)^2}}\cancel{(x-1)^2}(x+1) + \frac{C}{\cancel{(x+1)}}(x-1)^2\cancel{(x+1)}$$

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \longrightarrow (2)$$

$$1 = A(x^2 + \cancel{x} - \cancel{x} - 1) + B(x+1) + C((x)^2 + (1)^2 - 2(x)(1))$$

$$1 = A(x^2 - 1) + B(x+1) + C(x^2 + 1 - 2x)$$

$$1 = Ax^2 - A + Bx + B + Cx^2 + C - 2Cx$$

$$1 = Ax^2 + Cx^2 + Bx - 2Cx + B - A + C$$

$$0x^2 + 0x + 1 = (A+C)x^2 + (B-2C)x + (B-A+C)$$



Comparing coefficient of $x^2$, $x$ and Constant.

$$0 = A + C \longrightarrow (3)$$

$$0 = B - 2C \longrightarrow (4)$$

$$1 = B - A + C \longrightarrow (5)$$

put $x - 1 = 0$

$x = 1$ in eq (2)

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$1 = \underbrace{\cancel{A(1-1)(1+1)}}_{=0} + B(1+1) + C\underbrace{\cancel{(1-1)^2}}_{=0}$$

$$1 = B(2)$$

$$\boxed{\frac{1}{2} = B}$$

put $x+1=0$

$x=-1$ in equation #2.

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$1 = \underbrace{A(-1-1)(\cancel{-1+1})}_{=0} + \underbrace{B(\cancel{-1+1})}_{=0} + C(-1-1)^2$$

$$1 = C(-2)^2$$

$$1 = C(4)$$

$$\boxed{\frac{1}{4} = C}$$

put the value of "C" in eq 3



$$0 = A + C$$

$$0 = A + \frac{1}{4}$$

$$\boxed{-\frac{1}{4} = A}$$

put the value of "A", "B" and "C" in equation # 1.

$$\frac{1}{(x-1)^2(x+1)} = \frac{-\frac{1}{4}}{(x-1)} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{1}{4}}{(x+1)}$$

$$\boxed{\frac{1}{(x-1)^2(x+1)} = \frac{-1}{4(x-1)} + \frac{1}{2(x-1)^2} + \frac{1}{4(x+1)}}$$

Answer.

Question # 8 :- Resolve into partial fractions.

$$\frac{1}{(x^2-1)(x+1)}$$

Sol/

$$\frac{1}{(x^2-1)(x+1)}$$

put $x^2-1^2 = (x+1)(x-1)$

$$= \frac{1}{(x+1)(x-1)(x+1)}$$

$$= \frac{1}{(x-1)(x+1)^2}$$

Now Solve.



$$\frac{1}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \longrightarrow (1)$$

Multiply $(x-1)(x+1)^2$ on both sides.

$$\frac{1}{\cancel{(x-1)(x+1)^2}}\cancel{(x-1)(x+1)^2} = \frac{A}{\cancel{(x-1)}}\cancel{(x-1)}(x+1)^2$$

$$+ \frac{B}{\cancel{(x+1)}}(x-1)(x+1)^2$$

$$+ \frac{C}{\cancel{(x+1)^2}}(x-1)\cancel{(x+1)^2}$$

$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \longrightarrow (2)$$

$$1 = A\left((x)^2+(1)^2+2(x)(1)\right) + B\left(x^2+\cancel{x}-\cancel{x}-1\right) + C(x-1)$$

$$1 = A(x^2+1+2x) + B(x^2-1) + C(x-1)$$

$$1 = Ax^2 + A + 2Ax + Bx^2 - B + Cx - C$$

$$1 = Ax^2 + Bx^2 + 2Ax + Cx + A - B - C$$

$$0x^2 + 0x + 1 = (A+B)x^2 + (2A+C)x + (A-B-C)$$

Comparing Coefficent of $x^2$, $x$ and Constant

$$0 = A + B \longrightarrow (3)$$

$$0 = 2A + C \longrightarrow (4)$$

$$1 = A - B - C \longrightarrow (5)$$

put

$x - 1 = 0$

$x = 1$ in equation # (2)



$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$1 = A(1+1)^2 + B\underbrace{(1-1)}_{=0}(1+1) + C\underbrace{(1-1)}_{=0}$$

$$1 = A(2)^2$$

$$1 = A(4)$$

$$\boxed{\frac{1}{4} = A}$$

put

$x + 1 = 0$

$x = -1$ in equation # (2)

$$1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$1 = A\underbrace{(-1+1)}_{=0}^2 + B(-1-1)\underbrace{(-1+1)}_{=0} + C(-1-1)$$

$1 = C(-2)$

$\frac{1}{-2} = C$

$\boxed{-\frac{1}{2} = C}$

put the value of "A" in eq (3)

$0 = A + B$

$0 = \frac{1}{4} + B$

$\boxed{-\frac{1}{4} = B}$

put the value of A, B and C

in eq (1)



$$\frac{1}{(x-1)(x+1)^2} = \frac{\frac{1}{4}}{(x-1)} + \frac{-\frac{1}{4}}{(x+1)} + \frac{-\frac{1}{2}}{(x+1)^2}$$

$$\frac{1}{(x-1)(x+1)^2} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$

$$\boxed{\frac{1}{(x^2-1)(x+1)} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}}$$

Answer

we put $(x-1)(x+1)^2$

$= (x-1)(x+1)(x+1)$

$= (x^2-1)(x+1)$

# EXERCISE # 4.3

## Question # 1 , 6 , 8

**Question #1:-** Resolve into partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)}$$

Sol//

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{(Bx+C)}{(x^2+1)} \longrightarrow (1)$$

Multiply $(x+3)(x^2+1)$ on both sides

$$\frac{3x-11}{\cancel{(x+3)}\cancel{(x^2+1)}}\cancel{(x+3)}\cancel{(x^2+1)} = \frac{A}{\cancel{(x+3)}}\cancel{(x+3)}(x^2+1) + \frac{(Bx+C)}{\cancel{(x^2+1)}}(x+3)\cancel{(x^2+1)}$$



$$3x-11 = A(x^2+1) + (Bx+C)(x+3) \longrightarrow (2)$$

$$3x-11 = A(x^2+1) + Bx^2 + 3Bx + Cx + 3C$$

$$3x-11 = Ax^2 + A + Bx^2 + 3Bx + Cx + 3C$$

$$3x-11 = Ax^2 + Bx^2 + 3Bx + Cx + A + 3C$$

$$0x^2 + 3x - 11 = (A+B)x^2 + (3B+C)x + (A+3C)$$

Comparing coefficient of $x^2$, $x$ & Constant

$$0 = A+B \longrightarrow (3)$$
$$3 = 3B+C \longrightarrow (4)$$
$$-11 = A+3C \longrightarrow (5)$$

put $x+3=0$

$x=-3$ in equation # 2

$$3x-11 = A(x^2+1)+(Bx+C)(x+3)$$

$$3(-3)-11 = A\left((-3)^2+1\right)+\cancel{(B(-3)+C)(-3+3)}$$

$= 0$

$$-9-11 = A(9+1)$$

$$-20 = A(10)$$

$$\frac{-20}{10} = A$$

$$\boxed{-2 = A}$$

put the value of "A" in equation # 3

$$0 = A+B$$



$$0 = -2+B$$

$$\boxed{2 = B}$$

put the value of "B" in equation # 4

$$3 = 3B+C$$

$$3 = 3(2)+C$$

$$3 = 6+C$$

$$3-6 = C$$

$$\boxed{-3 = C}$$

put the value of "A", "B" & "C" in eq (1)

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{(x+3)}+\frac{(2x+(-3))}{(x^2+1)}$$

$$\boxed{\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{(x+3)}+\frac{2x-3}{x^2+1}}$$ Answer.

## Question # 6 :- Resolve into partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)}$$

Sol

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+4)} \longrightarrow (1)$$

Multiply $(x+2)(x^2+4)$ on both sides.

$$\frac{x^2}{\cancel{(x+2)}\cancel{(x^2+4)}}\cancel{(x+2)}\cancel{(x^2+4)} = \frac{A}{\cancel{(x+2)}}\cancel{(x+2)}(x^2+4) + \frac{(Bx+C)}{\cancel{(x^2+4)}}(x+2)\cancel{(x^2+4)}$$

$$x^2 = A(x^2+4) + (Bx+C)(x+2) \longrightarrow (2)$$

$$x^2 = Ax^2 + 4A + Bx^2 + 2Bx + Cx + 2C$$

$$x^2 = Ax^2 + Bx^2 + 2Bx + Cx + 4A + 2C$$



$$1x^2 + 0x + 0 = (A+B)x^2 + (2B+C)x + (4A+2C)$$

Comparing Coefficient of $x^2$, $x$ & Costant.

$$1 = A + B \longrightarrow (3)$$

$$0 = 2B + C \longrightarrow (4)$$

$$0 = 4A + 2C \longrightarrow (5)$$

Put $x+2=0$

$x = -2$ in eq, # (2)

$$x^2 = A(x^2+4) + (Bx+C)(x+2)$$

$$(-2)^2 = A\left((-2)^2+4\right) + \left(B(-2)+C\right)\underbrace{(-2+2)}_{=0}$$

$$4 = A(4+4)$$

$$4 = A(8)$$

$$\frac{4}{8} = A$$

$$\boxed{\frac{1}{2} = A}$$

put the value of "A" in eq #3

$$1 = A + B$$

$$1 = \frac{1}{2} + B$$

$$1 - \frac{1}{2} = B$$

$$\frac{1}{1} - \frac{1}{2} = B$$



$$\frac{2-1}{2} = B$$

$$\boxed{\frac{1}{2} = B}$$

put the value of "B" in eq #4

$$0 = 2B + C$$

$$0 = \cancel{2}\left(\frac{1}{\cancel{2}}\right) + C$$

$$0 = 1 + C$$

$$\boxed{-1 = C}$$

Put the value of "A", "B" & "C" in equation # 1.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+4)}$$

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1/2}{(x+2)} + \frac{\left(\left(\frac{1}{2}\right)x+(-1)\right)}{(x^2+4)}$$

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{\left(\frac{1x-2}{2}\right)}{(x^2+4)}$$ → LCM

$$\boxed{\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}}$$ Answer.

**Question # 8:-** Resolve into Partial fractions.

$$\frac{x^2+1}{x^3+1}$$

Sol// We know that



$$\boxed{x^3+1 = (x+1)(x^2-x+1)}$$

By using this formula $a^3+b^3 = (a+b)(a^2-ab+b^2)$

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2-x+1)} \longrightarrow (1)$$

Multiply $(x+1)(x^2-x+1)$ on both sides

$$\frac{x^2+1}{\cancel{(x+1)(x^2-x+1)}}\cancel{(x+1)(x^2-x+1)} = \frac{A}{\cancel{(x+1)}}\cancel{(x+1)}(x^2-x+1) + \frac{(Bx+C)}{\cancel{(x^2-x+1)}}(x+1)\cancel{(x^2-x+1)}$$

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1) \longrightarrow (2)$$

$$x^2+1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$$

$$x^2+1 = Ax^2+Bx^2+Bx-Ax+Cx+A+C$$

$$1x^2+0x+1 = (A+B)x^2+(B-A+C)x+(A+C)$$

Comparing Coefficient of $x^2$, $x$ & Constant.

$$1 = A+B \longrightarrow (3)$$

$$0 = B-A+C \longrightarrow (4)$$

$$1 = A+C \longrightarrow (5)$$

put $x+1=0$

$x=-1$ in eq # (2)

$$x^2+1 = A(x^2-x+1)+(Bx+C)(x+1)$$



$$(-1)^2+1 = A((-1)^2-(-1)+1)+\cancel{(B(-1)+C)(-1+1)}$$

$=0$

$$1+1 = A(1+1+1)$$

$$2 = A(3)$$

$$\boxed{2/3 = A}$$

Put the value of "A" in eq # (3)

$$1 = A+B$$

$$1 = \frac{2}{3}+B$$

$$1-\frac{2}{3} = B \Rightarrow \frac{3-2}{3} = B$$

$$\boxed{\frac{1}{3} = B}$$

Put the value of "A" in eq # (5)

$$1 = A + C$$

$$1 = \frac{2}{3} + C$$

$$1 - \frac{2}{3} = C$$

$$\frac{3-2}{3} = C$$

$$\boxed{\frac{1}{3} = C}$$

Now put the value of "A", "B" & "C" in equation # 1.

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)}$$



$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{\frac{2}{3}}{(x+1)} + \frac{\frac{1}{3}x + \frac{1}{3}}{(x^2-x+1)}$$

LCM

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{2}{3(x+1)} + \frac{\frac{1x+1}{3}}{(x^2-x+1)}$$

Also put $(x+1)(x^2-x+1) = x^3+1$

$$\boxed{\frac{x^2+1}{x^3+1} = \frac{2}{3(x+1)} + \frac{x+1}{3(x^2-x+1)}}$$

Answer

# EXERCISE # 4.4

## Question # 3, 6

Resolve into Partial fractions.

**Question # 3:-** $\dfrac{x^2}{(x+1)(x^2+1)^2}$

Sol

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2+1)} + \frac{(Dx+E)}{(x^2+1)^2} \longrightarrow (1)$$

Multiply $(x+1)(x^2+1)^2$ on both sides.

$$\frac{x^2}{\cancel{(x+1)(x^2+1)^2}}\cancel{(x+1)(x^2+1)^2} = \frac{A}{\cancel{(x+1)}}\cancel{(x+1)}(x^2+1)^2$$
$$+ \frac{(Bx+C)}{\cancel{(x^2+1)}}(x+1)(x^2+1)^{\cancel{2}}$$
$$+ \frac{(Dx+E)}{\cancel{(x^2+1)^2}}(x+1)\cancel{(x^2+1)^2}$$



$$x^2 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1) + (Dx+E)(x+1) \longrightarrow (2)$$

$$x^2 = A\left[(x^2)^2 + (1)^2 + 2(x^2)(1)\right] + (Bx+C)\left[x^3 + x + x^2 + 1\right]$$
$$+ (Dx+E)\left[x+1\right]$$

$$x^2 = A\left[x^4 + 1 + 2x^2\right] + Bx\left[x^3 + x + x^2 + 1\right] + C\left[x^3 + x + x^2 + 1\right]$$
$$+ Dx\left[x+1\right] + E\left[x+1\right]$$

$$x^2 = Ax^4 + A + 2Ax^2 + Bx^4 + Bx^2 + Bx^3 + Bx + Cx^3 + Cx$$
$$+ Cx^2 + C + Dx^2 + Dx + Ex + E$$

$$0x^4 + 0x^3 + 1x^2 + 0x + 0 = Ax^4 + Bx^4 + Bx^3 + Cx^3 + 2Ax^2 + Bx^2 + Cx^2$$
$$+ Dx^2 + Bx + Cx + Dx + Ex + A + C + E$$

$$0x^4+0x^3+1x^2+0x+0 = (A+B)x^4+(B+C)x^3+(2A+B+C+D)x^2$$
$$+(B+C+D+E)x+(A+C+E)$$

Comparing Coefficient of $x^4, x^3, x^2, x$ & Constant

$$0 = A+B \longrightarrow (3)$$
$$0 = B+C \longrightarrow (4)$$
$$1 = 2A+B+C+D \longrightarrow (5)$$
$$0 = B+C+D+E \longrightarrow (6)$$
$$0 = A+C+E \longrightarrow (7)$$

put $x+1=0$

$x=-1$ in eq (2)

$$x^2 = A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)+(Dx+E)(x+1)$$

$$(-1)^2 = A\left((-1)^2+1\right)^2+\underbrace{\cancel{(B(-1)+C)(-1+1)((-1)^2+1)}}_{=0}+\underbrace{\cancel{(D(-1)+E)(-1+1)}}_{=0}$$

$$1 = A(1+1)^2$$

$$1 = A(2)^2$$

$$1 = A(4)$$

$$\boxed{\frac{1}{4} = A}$$

put the value of "A" in eq # 3

$$0 = A+B$$

$$0 = \frac{1}{4}+B$$

$$\boxed{-\frac{1}{4} = B}$$

put the value of "B" in equation # 4

$$0 = B + C$$

$$0 = -\frac{1}{4} + C$$

$$\boxed{\frac{1}{4} = C}$$

put the value of A, B & C in eq #5

$$1 = 2A + B + C + D$$

$$1 = 2\left(\frac{1}{4}\right) + \left(-\frac{1}{4}\right) + \left(\frac{1}{4}\right) + D$$

$$1 = \frac{2}{4} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} + D$$

$$\frac{1}{1} - \frac{2}{4} = D$$



$$\frac{4-2}{4} = D$$

$$\frac{2}{4} = D \qquad \text{Divided by "2"}$$

$$\boxed{\frac{1}{2} = D}$$

put the value of B & D in eq # 6

$$0 = B + D + E + C$$

$$0 = -\cancel{\frac{1}{4}} + \frac{1}{2} + E + \cancel{\frac{1}{4}}$$

$$\boxed{-\frac{1}{2} = E}$$

Put the value of A, B, C, D & E in eq #1

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{\frac{1}{4}}{(x+1)} + \frac{-\frac{1}{4}x+\frac{1}{4}}{(x^2+1)} + \frac{\frac{1}{2}x+\left(-\frac{1}{2}\right)}{(x^2+1)^2}$$

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} + \frac{\frac{-x+1}{4}}{x^2+1} + \frac{\frac{x-1}{2}}{(x^2+1)^2}$$

$$\boxed{\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} + \frac{-x+1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}}$$



Answer

OR

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{1}{4(x+1)} - \frac{(x-1)}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Q#6:- Resolve into Partial fraction.

$$\frac{x^5}{(x^2+1)^2}$$

Sol//

$$\frac{x^5}{(x^2+1)^2}$$

$$= \frac{x^5}{(x^2)^2+(1)^2+2(x^2)(1)}$$

$$= \frac{x^5}{x^4+1+2x^2} \quad \text{improper fraction}$$

Now Converted into proper fraction

$$\begin{array}{r|l} & x \\ x^4+1+2x^2 & x^5 \\ & \underline{\cancel{x^5} + x + 2x^3} \\ & \quad -\;\;\; - \;\;\;\; - \\ & -x-2x^3 \end{array}$$

$$\frac{x^5}{x^4} = x$$



$$\frac{x^5}{x^4+1+2x^2} = x + \frac{-x-2x^3}{x^4+1+2x^2}$$

$$= x + \frac{-1(x+2x^3)}{x^4+1+2x^3}$$

put $x^4+1+2x^3 = (x^2+1)^2$

$$= x - \frac{(x+2x^3)}{(x^2+1)^2} \longrightarrow (1)$$

First we solve

$$\frac{x+2x^3}{(x^2+1)^2} = \frac{(Ax+B)}{(x^2+1)} + \frac{(Cx+D)}{(x^2+1)^2} \longrightarrow 2$$

Multiply $(x^2+1)^2$ on both sides.

$$\frac{x+2x^3}{\cancel{(x^2+1)^2}}\cancel{(x^2+1)^2} = \frac{(Ax+B)}{\cancel{(x^2+1)}}(x^2+1)^2 + \frac{(Cx+D)}{\cancel{(x^2+1)^2}}\cancel{(x^2+1)^2}$$

$$x+2x^3 = (Ax+B)(x^2+1) + (Cx+D)$$

$$x+2x^3 = Ax^3 + Ax + Bx^2 + B + Cx + D$$

$$2x^3 + 0x^2 + 1x + 0 = Ax^3 + Bx^2 + Ax + Cx + B + D$$

$$2x^3 + 0x^2 + 1x + 0 = Ax^3 + Bx^2 + (A+C)x + (B+D)$$



Comparing Coefficient of $x^3, x^2, x$ & Constant.

$\boxed{2 = A} \longrightarrow 3$

$\boxed{0 = B} \longrightarrow 4$

$1 = A + C \longrightarrow 5$

$0 = B + D \longrightarrow 6$

put $A=2$ in eq #5

$1 = A + C$

$1 = 2 + C$

$1 - 2 = C$

$\boxed{-1 = C}$

put $B=0$ in eq #6

$$0 = B + D$$

$$0 = 0 + D$$

$$\boxed{0 = D}$$

put the value of A, B, C & D in eq# 2

$$\frac{x+2x^3}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$$

$$\frac{x+2x^3}{(x^2+1)^2} = \frac{2x+0}{x^2+1} + \frac{-1x+0}{(x^2+1)^2}$$

$$\frac{x+2x^3}{(x^2+1)^2} = \frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2}$$



Now put the value of $\frac{x+2x^3}{(x^2+1)^2}$ in eq ①.

$$\frac{x^5}{(x^2+1)^2} = x - \frac{x+2x^3}{(x^2+1)^2}$$

$$\frac{x^5}{(x^2+1)^2} = x - \left(\frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2}\right)$$

$$\boxed{\frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}}$$

Answer.

# MISCELLANEOUS EXERCISE # 4

**Question # 1.** MCQs (Doyourself)

**Question # 2.** Short answers

**Q#2(i):- Define a rational fraction.**

Ans:- An expression of the form $\frac{N(x)}{D(x)}$, where $N(x)$ and $D(x)$ are polynomial in "$x$" with real co-efficients, is called rational fraction.



**Q#2(ii):- What is an improper fraction?**

Ans:- A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction if degree of $N(x)$ is greater than degree of $D(x)$.

**Q#2(iii):- What is a proper fraction?**

Ans:- A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called a proper fraction if degree of the polynomial $N(x) <$ (less than) the degree of the polynomial $D(x)$.

**Q#2 (iv):- What are partial fraction.**

**Ans:-** A single fraction written in the forms of its components is said to be resolved into partial fraction.

**Q#2 (v):- How can we make partial fractions of $\frac{x-2}{(x+2)(x+3)}$ ?**



**Sol**

$$\frac{x-2}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)} \longrightarrow (1)$$

Multiply $(x+2)(x+3)$ on both sides.

$$\frac{x-2}{\cancel{(x+2)(x+3)}}\cancel{(x+2)(x+3)} = \frac{A}{\cancel{(x+2)}}\cancel{(x+2)}(x+3) + \frac{B}{\cancel{(x+3)}}(x+2)\cancel{(x+3)}$$

$$x-2 = A(x+3) + B(x+2) \longrightarrow (2)$$

put $x+2=0$

$x=-2$ in eq (2)

$$-2-2 = A(-2+3) + B(\underbrace{-2+2}_{=0})$$

$$-4 = A(1)$$

$$\boxed{-4 = A}$$

put $x+3=0$

$x=-3$ in eq # 2

$$x-2 = A(x+3) + B(x+2)$$

$$-3-2 = A\underbrace{\cancel{(-3+3)}}_{=0} + B(-3+2)$$

$$-5 = B(-1)$$

$$\frac{-5}{-1} = B$$

$$\boxed{5 = B}$$

Now put the value of "A" and "B" in equation # 1



$$\frac{x-2}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$$

$$\boxed{\frac{x-2}{(x+2)(x+3)} = \frac{-4}{x+2} + \frac{5}{x+3}}$$

Answer.

# Unit # 5

# SETS AND FUNCTIONS

## EXERCISE # 5.1

Q#1 (i, ii, iii, iv) , Q#3 (i, ii, iii, iv, v, vi)

Q#4 (i, iii) , Q#6 (i, ii)

**Question #1 :-** If $X = \{1, 4, 7, 9\}$ and $Y = \{2, 4, 5, 9\}$ Then find:

(i) $X \cup Y$ (ii) $X \cap Y$

(iii) $Y \cup X$ (iv) $Y \cap X$

Sol//



(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$X \cup Y = \{1, 2, 4, 5, 7, 9\}$ Answer.

(ii) $X \cap Y = \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\}$

$X \cap Y = \{4, 9\}$ Answer.

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\}$

$Y \cup X = \{1, 2, 4, 5, 7, 9\}$ Answer.

(iv) $Y \cap X = \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\}$

$Y \cap X = \{4, 9\}$ Answer.

**Question # 3:-** If $X = \phi$ , $Y = Z^+$, $T = O^+$ ,
then find :

(i) $X \cup Y$ (ii) $X \cup T$ (iii) $Y \cup T$

(iv) $X \cap Y$ (v) $X \cap T$ (vi) $Y \cap T$

Sol//

$X = \phi$

$X = \{ \}$

$Y = Z^+$

$Y = \{0, 1, 2, 3, \ldots\}$

$T = O^+$

$T = \{1, 3, 5, 7, \ldots\}$

(i) $X \cup Y = \{ \} \cup \{0, 1, 2, 3, \ldots\}$

$X \cup Y = \{0, 1, 2, 3, \ldots\}$ Answer



(ii) $X \cup T = \{ \} \cup \{1, 3, 5, 7, \ldots\}$

$X \cup T = \{1, 3, 5, 7, \ldots\}$ Answer

(iii) $Y \cup T = \{0, 1, 2, 3, \ldots\} \cup \{1, 3, 5, 7, \ldots\}$

$Y \cup T = \{0, 1, 2, 3, 4, 5, \ldots\}$ Answer

(iv) $X \cap Y = \{ \} \cap \{0, 1, 2, 3, \ldots\}$

$X \cap Y = \{ \}$ Answer

(v) $X \cap T = \{ \} \cap \{1, 3, 5, 7, \ldots\}$

$X \cap T = \{ \}$ Answer

(vi) $Y \cap T = \{0, 1, 2, 3, \ldots\} \cap \{1, 3, 5, 7, \ldots\}$

$Y \cap T = \{1, 3, 5, 7, \ldots\}$ Answer

**Question #4:-** If $U=\{x \mid x \in N \wedge 3<x\leq 25\}$,

$X=\{x \mid x \text{ is prime} \wedge 8<x<25\}$ and

$Y=\{x \mid x \in W \wedge 4\leq x\leq 17\}$

Find the value of (i) $(X \cup Y)'$, (iii) $(X \cap Y)'$

Sol//

$U=\{x \mid x \in N \wedge 3<x\leq 25\}$

$U=\{4,5,6,\ldots\ldots 25\}$

$X=\{x \mid x \text{ is prime} \wedge 8<x<25\}$

$X=\{11,13,17,19,23\}$

$Y=\{x \mid x \in W \wedge 4\leq x\leq 17\}$



$Y=\{4,5,6,\ldots\ldots 17\}$

(i) $(X \cup Y)'=?$

$(X \cup Y)'=U-(X \cup Y)$

$(X \cup Y)'=\{4,5,6\ldots\ldots 25\}-(\{11,13,17,19,23\} \cup \{4,5,6,\ldots\ldots 17\})$

$(X \cup Y)'=\{4,5,6,\ldots\ldots 25\}-\{4,5,6,\ldots\ldots 17,19,23\}$

$(X \cup Y)'=\{18,20,21,22,24,25\}$ Answer

(ii) $(X \cap Y)'=?$

$(X \cap Y)'=U-(X \cap Y)$

$(X \cap Y)'=\{4,5,6,\ldots\ldots 25\}-(\{11,13,17,19,23\} \cap \{4,5,6\ldots\ldots 17\})$

$(X \cap Y)'=\{4,5,6,\ldots\ldots 25\}-\{11,13,17\}$

$(X \cap Y)'=\{4,5,6,7,8,9,10,12,14,15,16,18,19\ldots\ldots 25\}$ Answer.

**Question#6:** If $A = N$ and $B = W$

then find the value of

(i) $A - B$ (ii) $B - A$

Sol/

$A = N$

$A = \{1, 2, 3, 4, \ldots\ldots\}$

$B = W$

$B = \{0, 1, 2, 3, 4, \ldots\ldots\}$

(i) $A - B = ?$

$A - B = \{1, 2, 3, 4, \ldots\ldots\} - \{0, 1, 2, 3, 4, \ldots\ldots\}$

$A - B = \{\ \}$ Answer



(ii) $B - A = ?$

$B - A = \{0, 1, 2, 3, 4, \ldots\ldots\} - \{1, 2, 3, 4, \ldots\}$

$B - A = \{0\}$ Answer

—— xx —————— xx ——

When we solve the Questions of Subtraction of Sets, then only write the elements of first set, that are not present in second set.

# EXERCISE # 5.2

Question # 1 (v, vi, vii, viii)

Question # 2 (iv), Question # 3

Question # 4 (ii)

**Question # 1:-** If $X = \{1, 3, 5, 7, \ldots, 19\}$

$Y = \{0, 2, 4, 6, 8, \ldots, 20\}$ and

$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

then find the following :

(v) $X \cup (Y \cap Z) = ?$



Sol/

$X \cup (Y \cap Z) = \{1, 3, 5, 7, \ldots, 19\} \cup (\{0, 2, 4, 6, 8, \ldots, 20\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\})$

$X \cup (Y \cap Z) = \{1, 3, 5, 7, \ldots, 19\} \cup \{2\}$

$X \cup (Y \cap Z) = \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ Answer.

(vi) $(X \cup Y) \cap (X \cup Z) = ?$

Sol/

$(X \cup Y) \cap (X \cup Z) = (\{1, 3, 5, 7, \ldots 19\} \cup \{0, 2, 4, 6, 8, \ldots 20\}) \cap (\{1, 3, 5, 7, \ldots 19\} \cup \{2, 3, 5, 7, \ldots 23\})$

$(X \cup Y) \cap (X \cup Z) = \{0, 1, 2, 3, 4, 5, \ldots 19, 20\} \cap \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$

$(X \cup Y) \cap (X \cup Z) = \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$ Answer.

(vii) $X\cap(Y\cup Z)=?$

Sol//

$$X\cap(Y\cup Z)=\{1,3,5,7,\ldots,19\}\cap(\{0,2,4,6,8,\ldots,20\}\cup\{2,3,5,7,11,13,17,19,23\})$$

$$X\cap(Y\cup Z)=\{1,3,5,7,\ldots,19\}\cap\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$$

$$X\cap(Y\cup Z)=\{3,5,7,11,13,17,19\}$$ Answer.

(viii) $(X\cap Y)\cup(X\cap Z)=?$

Sol//

$$(X\cap Y)\cup(X\cap Z)=(\{1,3,5,7,\ldots,19\}\cap\{0,2,4,6,8,\ldots,20\})\cup(\{1,3,5,7,\ldots,19\}\cap\{2,3,5,7,11,13,17,19,23\})$$

$$(X\cap Y)\cup(X\cap Z)=\{\ \}\cup\{3,5,7,11,13,17,19\}$$



$$(X\cap Y)\cup(X\cap Z)=\{3,5,7,11,13,17,19\}$$ Answer.

Question#2 :-

If $A=\{1,2,3,4,5,6\}$, $B=\{2,4,6,8\}$, $C=\{1,4,8\}$.

Prove the following identities.

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Sol// L.H.S $=A\cup(B\cap C)$

$$A\cup(B\cap C)=\{1,2,3,4,5,6\}\cup(\{2,4,6,8\}\cap\{1,4,8\})$$

$$A\cup(B\cap C)=\{1,2,3,4,5,6\}\cup\{4,8\}$$

$$A\cup(B\cap C)=\{1,2,3,4,5,6,8\}\longrightarrow (1)$$

$R.H.S = (A \cup B) \cap (A \cup C)$

$(A \cup B) \cap (A \cup C) = (\{1,2,3,4,5,6\} \cup \{2,4,6,8\}) \cap (\{1,2,3,4,5,6\} \cup \{1,4,8\})$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,8\} \cap \{1,2,3,4,5,6,8\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,8\} \longrightarrow (2)$

from equation (1) and (2)

$L.H.S = R.H.S$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**Question #3:-** If $U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,3,5,7,9\}$ , $B = \{2,3,5,7\}$

Then verify the De-Morgan's Laws.



(i) $(A \cap B)' = A' \cup B'$

Sol//

$L.H.S = (A \cap B)'$

$(A \cap B)' = U - (A \cap B)$

$(A \cap B)' = \{1,2,3,4,5,6,7,8,9,10\} - (\{1,3,5,7,9\} \cap \{2,3,5,7\})$

$(A \cap B)' = \{1,2,3,4,5,6,7,8,9,10\} - \{3,5,7\}$

$(A \cap B)' = \{1,2,4,6,8,9,10\} \longrightarrow (1)$

$R.H.S = A' \cup B'$

$A' \cup B' = (U - A) \cup (U - B)$

$A' \cup B' = (\{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}) \cup (\{1,2,3,4,5,6,7,8,9,10\} - \{2,3,5,7\})$

$A'\cup B' = \{2,4,6,8,10\} \cup \{1,4,6,8,9,10\}$

$A'\cup B' = \{1,2,4,6,8,9,10\} \longrightarrow 2$

From equation 1 & 2

L.H.S = R.H.S

$(A\cap B)' = A'\cup B'$

**Q#3(ii):- Verify De-Morgan's Law**

$(A\cup B)' = A'\cap B'$

Sol

L.H.S = $(A\cup B)'$

$(A\cup B)' = U-(A\cup B)$



$(A\cup B)' = \{1,2,3,4,5,6,7,8,9,10\} - (\{1,3,5,7,9\}\cup\{2,3,5,7\})$

$(A\cup B)' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,5,7,9\}$

$(A\cup B)' = \{4,6,8,10\} \longrightarrow 1$

R.H.S = $A'\cap B'$

$A'\cap B' = (U-A)\cap(U-B)$

$A'\cap B' = (\{1,2,3,4,5,6,7,8,9,10\}-\{1,3,5,7,9\})\cap(\{1,2,3,4,5,6,7,8,9,10\}-\{2,3,5,7\})$

$A'\cap B' = \{2,4,6,8,10\} \cap \{1,4,6,8,9,10\}$

$A'\cap B' = \{4,6,8,10\} \longrightarrow 2$

From equation 1 & 2

L.H.S = R.H.S

$(A\cup B)' = A'\cap B'$

**Question #4:** If $U=\{1,2,3,\ldots\ldots 20\}$

$X=\{1,3,7,9,15,18,20\}$ and

$Y=\{1,3,5,\ldots\ldots,17\}$ then show that

(ii) $Y-X = Y\cap X'$

Sol// L.H.S $= Y-X$

$Y-X = \{1,3,5,7,9,11,13,15,17\}-\{1,3,7,9,15,18,20\}$

$Y-X = \{5,11,13,17\} \longrightarrow (1)$

R.H.S $= Y\cap X'$



$Y\cap X' = Y\cap(U-X')$

$Y\cap X' = \{1,3,5,7,9,11,13,15,17\}\cap\left(\{1,2,3,\ldots,20\}-\{1,3,7,9,15,18,20\}\right)$

$Y\cap X' = \{1,3,5,7,9,11,13,15,17\}\cap\{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$Y\cap X' = \{5,11,13,17\} \longrightarrow (2)$

From equation # (1) & (2)

L.H.S = R.H.S

$Y-X = Y\cap X'$

Hence proved

# EXERCISE # 5.3

Question # 1 (i, iii, v) , Question # 2 (ii, iii)

Question # 4 (iii, v)

**Question # 1 :-** If $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$

$A = \{1, 3, 5, 7, 9\}$ , $B = \{1, 4, 7, 10\}$

then verify the following questions.

(i) $A - B = A \cap B'$

Sol/

$L.H.S = A - B$

$A - B = \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$



$A - B = \{3, 5, 9\} \longrightarrow (1)$

$R.H.S = A \cap B'$

$A \cap B' = A \cap (U - B)$

$A \cap B' = \{1, 3, 5, 7, 9\} \cap (\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 4, 7, 10\})$

$A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$

$A \cap B' = \{3, 5, 9\} \longrightarrow (2)$

From equation # (1) and (2)

$L.H.S = R.H.S$

$A - B = A \cap B'$ Hence proved

## Question # 1 (iii) :-

$$(A\cup B)' = A' \cap B'$$

Sol//

$L.H.S = (A\cup B)'$

$(A\cup B)' = U - (A\cup B)$

$(A\cup B)' = \{1,2,3,4,\ldots\ldots,10\} - (\{1,3,5,7,9\}\cup\{1,4,7,10\})$

$(A\cup B)' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,4,5,7,9,10\}$

$(A\cup B)' = \{2,6,8\} \longrightarrow (1)$

$R.H.S = A' \cap B'$

$A'\cap B' = (U-A)\cap(U-B)$

$A'\cap B' = (\{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}) \cap (\{1,2,3,4,5,6,7,8,9,10\} - \{1,4,7,10\})$



$A'\cap B' = \{2,4,6,8,10\} \cap \{2,3,5,6,8,9\}$

$A'\cap B' = \{2,6,8\} \longrightarrow (2)$

From equation # (1) & (2)

$L.H.S = R.H.S$

$$(A\cup B)' = A'\cap B'$$

Hence proved

## Question # 1 (v) :-

$$(A-B)' = A' \cup B$$

Sol//

$L.H.S = (A-B)'$

$(A-B)' = U-(A-B)$

$(A-B)' = \{1,2,3,4,\ldots\ldots,10\} - (\{1,3,5,7,9\} - \{1,4,7,10\})$

$(A-B)' = \{1,2,3,4,5,6,7,8,9,10\} - \{3,5,9\}$

$(A-B)' = \{1,2,4,6,7,8,10\} \longrightarrow (1)$

$R.H.S = A' \cup B$

$A' \cup B = (U-A) \cup B$

$A' \cup B = (\{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}) \cup \{1,4,7,10\}$

$A' \cup B = \{2,4,6,8,10\} \cup \{1,4,7,10\}$

$A' \cup B = \{1,2,4,6,7,8,10\} \longrightarrow (2)$



from eq (1) & (2)

$L.H.S = R.H.S$

$(A-B)' = A' \cup B$

Hence proved
am.

**Question #2** :- If $U=\{1,2,3,4,\ldots\ldots,10\}$

$A=\{1,3,5,7,9\}$ ; $B=\{1,4,7,10\}$ ; $C=\{1,5,8,10\}$

then verify the following :

(ii) $(A\cap B)\cap C = A\cap(B\cap C)$

Sol// L.H.S $= (A\cap B)\cap C$

$(A\cap B)\cap C = \left(\{1,3,5,7,9\}\cap\{1,4,7,10\}\right)\cap\{1,5,8,10\}$

$(A\cap B)\cap C = \{1,7\}\cap\{1,5,8,10\}$

$(A\cap B)\cap C = \{1\} \longrightarrow (1)$

R.H.S $= A\cap(B\cap C)$ 

$A\cap(B\cap C) = \{1,3,5,7,9\}\cap\left(\{1,4,7,10\}\cap\{1,5,8,10\}\right)$

$A\cap(B\cap C) = \{1,3,5,7,9\}\cap\{1,10\}$

$A\cap(B\cap C) = \{1\} \longrightarrow (2)$

From equation # (1) & (2)

L.H.S = R.H.S

$(A\cap B)\cap C = A\cap(B\cap C)$

**Q#2(iii)** $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Sol// L.H.S $= A\cup(B\cap C)$

$A\cup(B\cap C) = \{1,3,5,7,9\}\cup\left(\{1,4,7,10\}\cap\{1,5,8,10\}\right)$

$A\cup(B\cap C) = \{1,3,5,7,9\}\cup\{1,10\}$

$A\cup(B\cap C) = \{1,3,5,7,9,10\} \longrightarrow (1)$

$R.H.S = (A\cup B)\cap(A\cup C)$

$(A\cup B)\cap(A\cup C) = (\{1,3,5,7,9\}\cup\{1,4,7,10\})\cap(\{1,3,5,7,9\}\cup\{1,5,8,10\})$

$(A\cup B)\cap(A\cup C) = \{1,3,4,5,7,9,10\}\cap\{1,3,5,7,8,9,10\}$

$(A\cup B)\cap(A\cup C) = \{1,3,5,7,9,10\} \longrightarrow (2)$

From equation # (1) and (2)

$L.H.S = R.H.S$

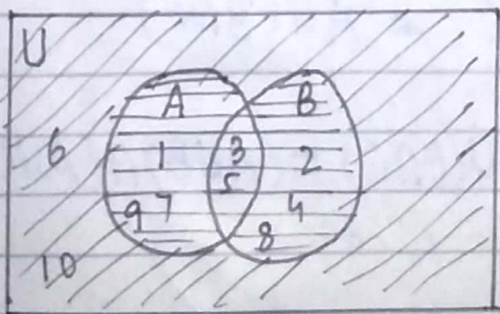
$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$



**Question #4:-** If $U=\{1,2,3,4,\ldots,10\}$, $A=\{1,3,5,7,9\}$ and $B=\{2,3,4,5,8\}$, then Prove the following questions by Venn diagram.

(iii) $(A\cup B)' = A'\cap B'$

Sol:



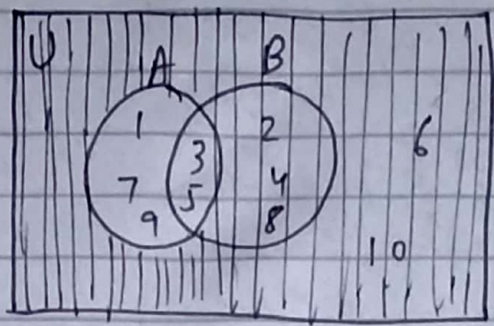

$A\cup B = ≡$

$U-(A\cup B) = ////$

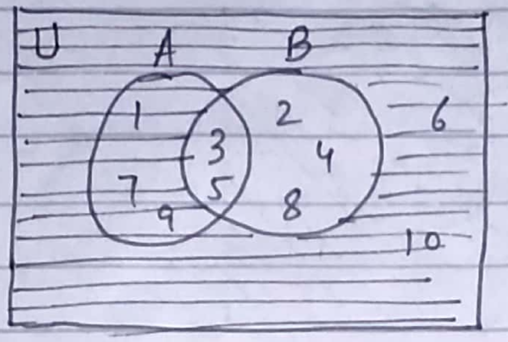
$L.H.S = (A\cup B)' = \{6,10\}$

R.H.S

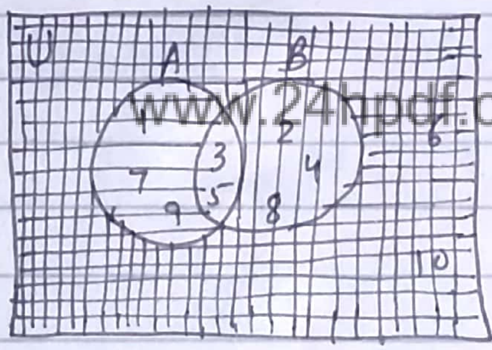
$= A' \cap B'$




$A' = U - A$




$B' = U - B$






$A' \cap B' = \{6, 10\}$

R.H.S

**L.H.S = R.H.S**

$\{6, 10\} = \{6, 10\}$

$(A \cup B)' = A' \cap B'$

Hence proved

## Q#4 (v):- $(A-B)' = A' \cup B$

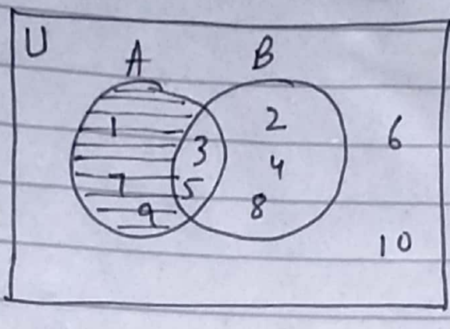
Sol//

$A = \{1, 3, 5, 7, 9\}$ , $B = \{2, 3, 4, 5, 8\}$

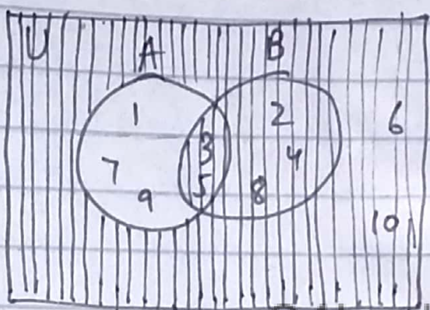
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

L.H.S

$= (A-B)'$



$A - B = \{1, 7, 9\}$



$(A-B)' = U - (A-B)$

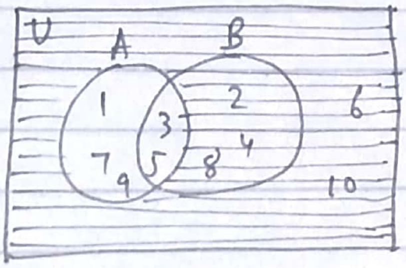
$= \{2, 4, 8, 3, 5, 6, 10\}$



L.H.S

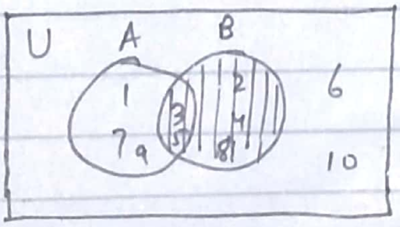
R.H.S

$= A' \cup B$

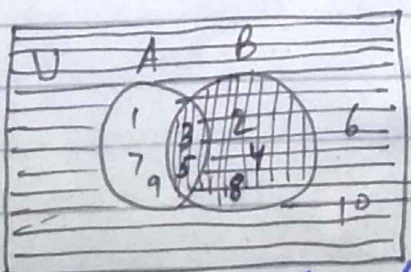


$A' = U - A$

$= \{2, 4, 6, 8, 10\}$



B



$A' \cup B = \{2, 4, 8, 3, 5, 6, 10\}$

L.H.S = R.H.S

$(A-B)' = A' \cup B$ Hence proved.

# EXERCISE # 5.4

## Question 3(iii), Question 5(ii)

**Question # 3(iii):-** Find a and b if

$$(3-2a, b-1) = (a-7, 2b+5)$$

Sol// $(3-2a, b-1) = (a-7, 2b+5)$

$3-2a = a-7$

$3+7 = a+2a$

$10 = 3a$

$\boxed{\frac{10}{3} = a}$ Answer

$b-1 = 2b+5$

$-1-5 = 2b-b$

$\boxed{-6 = b}$ Answer



**Question # 5(ii):-** If $X = \{a, b, c\}$ and $Y = \{d, e\}$, then find the number of elements in $Y \times X$

Sol//

Numbers of elements in $X = 3$

Numbers of elements in $Y = 2$

Numbers of elements in $Y \times X = 2 \times 3$

$= \boxed{6}$

Answer

# EXERCISE # 5.5

**Question # 3 (i, ii, iii), Question # 5 (ii, iii)**

**Question # 3:-** If $L = \{a, b, c\}$ and $M = \{d, e, f, g\}$, then find two binary relations in each:

(i) $L \times L = ?$

Sol// $L \times L = \{a, b, c\} \times \{a, b, c\}$

$L \times L = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$



(ii) $L \times M = ?$

$L \times M = \{a, b, c\} \times \{d, e, f, g\}$

$L \times M = \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)\}$

(iii) $M \times M = ?$

$M \times M = \{d, e, f, g\} \times \{d, e, f, g\}$

$M \times M = \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$

**Question # 5:-** If $L = \{x / x \in N \wedge x \leq 5\}$

$M = \{y / y \in P \wedge y < 10\}$ then make the following relations from L to M

(ii) $R_2 = \{(x, y) / y = x\}$

(iii) $R_3 = \{(x, y) / x + y = 6\}$

Sol//

$L = \{x / x \in N \wedge x \leq 5\}$

$L = \{1, 2, 3, 4, 5\}$

$M = \{y / y \in P \wedge y < 10\}$

$M = \{2, 3, 5, 7\}$



L to M = ? it means $L \times M = ?$

$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$

$L \times M = \{(1,2), (1,3), (1,5), (1,7),$
$(2,2), (2,3), (2,5), (2,7),$
$(3,2), (3,3), (3,5), (3,7),$
$(4,2), (4,3), (4,5), (4,7),$
$(5,2), (5,3), (5,5), (5,7)\}$

$R_2 = \{(x, y) / y = x\}$

$R_2 = \{(2,2), (3,3), (5,5)\}$

$R_3 = \{(x, y) / x + y = 6\}$

$R_3 = \{(1,5), (3,3), (4,2)\}$

# MISCELLANEOUS EXERCISE #5

**Question #1 (M.C.Qs) Doyourself.**

**Question #2 (Complete)**

**Q#2 (i):- Define a subset and give one example.**

Ans:- Set A is said to be a subset of a set B, denoted by $A \subseteq B$, if each elements of A is an elements of B.

For example, if $A = \{3,4,5\}$ and $B = \{2,3,4,5,6\}$ then A is subset of B.



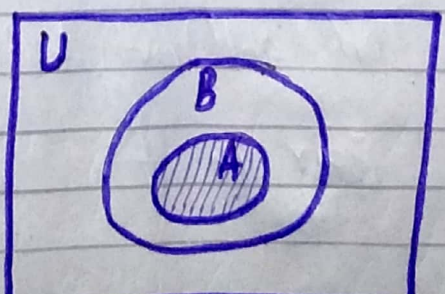
**Q#2 (ii):- Write all the subsets of the set $\{a,b\}$.**

Sol// let $A = \{a,b\}$

Subset of "A" are : $\phi, \{a\}, \{b\}, \{a,b\}$

**Q#2 (iii):- Show $A \cap B$ by Venn diagram when $A \subseteq B$.**

Sol//

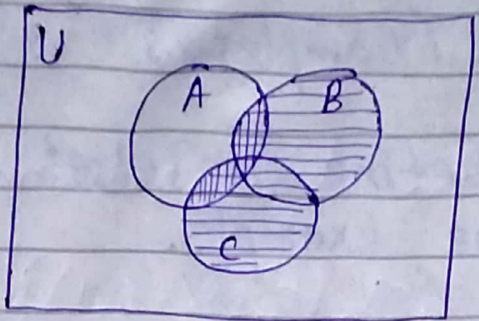



$A \cap B$
when $A \subseteq B$.

**Q#2(iv):-** Show by venn diagram

$A \cap (B \cup C)$.

Sol




$A \cap (B \cup C)$

**Q#2(v):-** Define intersection of two sets.

Ans:- The intersection of two sets A and B is written as $A \cap B$ and this set is consisting of all the common elements of A and B.



**Q#2(vi):-** Define a function.

Ans:- Suppose A and B are two non-empty sets, then relation $f: A \to B$ is called a function if

(i) $\text{Dom } f = A$

(ii) every $x \in A$ appears in one and only one ordered pair in $f$.

## Q#2 (vii):- Define one-one function.

**Ans:-** A function $f: A \to B$ is called one-one function, if all distinct elements of A have distinct images in B

i.e

$$f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2 \in A$$

OR

$$\forall \; x_1 \neq x_2 \in A$$



$$\Rightarrow f(x_1) \neq f(x_2)$$

## Q#2 (viii):- Define an onto function.

**Ans:-** A function $f: A \to B$ is called an onto function, if every elements of set B is an image of at least one element of set A.

i.e

$$\text{Range } f = B$$

**Q#2(ix) Define a bijective function.**

**Ans:-** A function $f: A \to B$ is called bijective function if function $f$ is one-one and onto.

$f = \{(1,2), (2,3), (3,4), (4,5)\}$.

This function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every elements of B is the images of at least one element of A.



**Q#2(x) Write De-Morgan's Laws.**

**Ans:-** (i) $(A \cap B)' = A' \cup B'$

(ii) $(A \cup B)' = A' \cap B'$

# Unit # 6

# BASIC STATISTICS

## EXERCISE # 6.1

Question # 1 , Question # 2

**Question # 1 :-** The following data shows the number of members in various families. Construct frequency distribution. Also find cumulative frequencies.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7.

Sol//



| Number of members | Talley marks | frequency | Commutative frequency |
|---|---|---|---|
| 2 | I | 1 | 1 |
| 3 | III | 3 | 1+3 = 4 |
| 4 | ~~IIII~~ I | 6 | 4+6 = 10 |
| 5 | IIII | 4 | 10+4 = 14 |
| 6 | III | 3 | 14+3 = 17 |
| 7 | ~~IIII~~ I | 6 | 17+6 = 23 |
| 8 | ~~IIII~~ | 5 | 23+5 = 28 |
| 9 | ~~IIII~~ I | 6 | 28+6 = 34 |
| 10 | II | 2 | 34+2 = 36 |
| 11 | II | 2 | 36+2 = 38 |
| 12 | I | 1 | 38+1 = 39 |
| | Total | 39 | |

## Question #3:-

From the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs. 100, 450, 500, 550, 580, 670, 1200, 1150, 1120, 950, 1130, 1230, 890, 780, 760, 670, 880, 890, 1050, 980, 970, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240.

(Hint: Make classes 450–549, 550–649, ......)

Sol//

| Class Limit | Talley Marks | Frequency |
|---|---|---|
| 450–549 | II | 2 |
| 550–649 | II | 2 |
| 650–749 | IIII | 4 |
| 750–849 | ~~IIII~~ | 5 |
| 850–949 | III | 3 |
| 950–1049 | IIII | 4 |
| 1050–1149 | ~~IIII~~ | 5 |
| 1150–1249 | ~~IIII~~ | 5 |
| | Total | 30 |

# EXERCISE # 6.2

## Question # 3 , 7 , 11 , 12 (smart course)

**Question # 3:-** Find arithmetic mean by direct method for the following set of data:

(i) 12, 14, 17, 20, 24, 29, 35, 45.

(ii) 200, 225, 350, 375, 270, 320, 290.

Sol/

(i) $A.M = \bar{X} = \frac{\Sigma x}{n}$

$$\bar{X} = \frac{12+14+17+20+24+29+35+45}{8}$$

$$\bar{X} = \frac{196}{8}$$

$\boxed{\bar{X} = 24.5}$ Answer.



(ii)

$$\bar{X} = \frac{200+225+350+375+270+320+290}{7}$$

$$\bar{X} = \frac{2030}{7}$$

$\boxed{\bar{X} = 290}$ Answer.

**Question # 7:-** The following data shows the number of children in various families. Find mode and median.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5.

Sol// Write the observations in ascending order.

2, 3, 3, 3, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 11, 11, 12.

**Mode:** The most frequent observation = **9, 4**

**Median:**

Number of observations = 38

Therefore, median is the mean of $19^{th}$ and $20^{th}$ observation

$$\text{Median} = \frac{7+7}{2}$$



$$\text{Median} = \frac{14}{2}$$

$$\text{Median} = \boxed{7}$$ Answer

**Question #11:-** On a vacation trip a family bought 21.3 liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter. Find the mean price paid per liter.

Sol//

| liters "x" | Price Per liter "p" | Amount "xp" |
|---|---|---|
| 21.3 | 39.90 | (21.3)(39.90) = 849.87 |
| 18.7 | 42.90 | (18.7)(42.90) = 802.23 |
| 23.5 | 40.90 | (23.5)(40.90) = 961.15 |
| $\Sigma x = 63.5$ | | $\Sigma xp = 2613.25$ |

$$\text{Mean Price} = \frac{\Sigma xp}{\Sigma x}$$

$$= \frac{2613.25}{63.5}$$

$$= \boxed{41.15} \text{ rupees per liter}$$

Answer
Jir.

**Question # 12:-** Calculate simple moving average of 3 years from the following data:

| Years | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|---|---|---|---|
| Values | 102 | 108 | 130 | 140 | 158 | 180 | 196 | 210 | 220 | 230 |

Sol//

| Years | Values | 3 Year moving Total | Average |
|---|---|---|---|
| 2001 | 102 | – | – |
| 2002 | 108 | 340 | 340/3 = 113.33 |
| 2003 | 130 | 378 | 378/3 = 126.00 |
| 2004 | 140 | 428 | 428/3 = 142.67 |
| 2005 | 158 | 478 | 478/3 = 159.33 |
| 2006 | 180 | 534 | 534/3 = 178.00 |
| 2007 | 196 | 586 | 586/3 = 195.33 |
| 2008 | 210 | 626 | 626/3 = 208.67 |
| 2009 | 220 | 660 | 660/3 = 220.00 |
| 2010 | 230 | – | |

Example

102+108+130 = 340

108+130+140 = 378

130+140+158 = 428

# EXERCISE # 6.3

## Question # 4, 5(ii), 7

**Question #4:-** The salaries of five teachers in Rupees are as follows.
11500, 12400, 15000, 14500, 14800.
Find Range and standard deviation.

**Sol//** $X = 11500, 12400, 15000, 14500, 14800$

Here $X_{max} = 15000$

$X_{min} = 11500$

$$Range = X_{max} - X_{min}$$

$$Range = 15000 - 11500$$



$$Range = \boxed{3500}$$ Answer.

(ii) **Standard Deviation = ?**

$$S.D = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$$

$$\bar{X} = \frac{\Sigma X}{n}$$

$$\bar{X} = \frac{11500+12400+15000+14500+14800}{5}$$

$$\bar{X} = \frac{68200}{5}$$

$$\boxed{\bar{X} = 13640}$$

| $X$ | $X-\bar{X}$ | $(X-\bar{X})^2$ |
|---|---|---|
| 11500 | $11500-13640 = -2140$ | $(-2140)^2 = 4579600$ |
| 12400 | $12400-13640 = -1240$ | $(-1240)^2 = 1537600$ |
| 15000 | $15000-13640 = 1360$ | $(1360)^2 = 1849600$ |
| 14500 | $14500-13640 = 860$ | $(860)^2 = 739600$ |
| 14800 | $14800-13640 = 1160$ | $(1160)^2 = 1345600$ |

$$\Sigma(x-\bar{x})^2 = 4579600 + 1537600 + 1849600 + 739600 + 1345600$$



$$= 1005200$$

**Formula**

$$S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$S.D = \sqrt{\frac{1005200}{5}}$$

$$S.D = \sqrt{2010400}$$

$$\boxed{S.D = 1417.88}$$

Answer

**Question #5 (ii):-** Find the standard deviation "S" of each set of numbers:

9, 3, 8, 8, 9, 8, 9, 18

Sol//

$X = 9, 3, 8, 8, 9, 8, 9, 18$

$$\bar{X} = \frac{\Sigma X}{n}$$

$$\bar{X} = \frac{9+3+8+8+9+8+9+18}{8}$$

$$\bar{X} = \frac{72}{8}$$

$$\boxed{\bar{X} = 9}$$



| $X$ | $X-\bar{X}$ | $(X-\bar{X})^2$ |
|---|---|---|
| 9 | $9-9 = 0$ | $(0)^2 = 0$ |
| 3 | $3-9 = -6$ | $(-6)^2 = 36$ |
| 8 | $8-9 = -1$ | $(-1)^2 = 1$ |
| 8 | $8-9 = -1$ | $(-1)^2 = 1$ |
| 9 | $9-9 = 0$ | $(0)^2 = 0$ |
| 8 | $8-9 = -1$ | $(-1)^2 = 1$ |
| 9 | $9-9 = 0$ | $(0)^2 = 0$ |
| 18 | $18-9 = 9$ | $(9)^2 = 81$ |
| | | |

$$\Sigma(X-\bar{X})^2 = 0+36+1+1+0+1+0+81$$

$$= 120$$

Formula

$$S.D = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$S.D = \sqrt{\frac{120}{8}}$$

$$S.D = \sqrt{15}$$

$$\boxed{S.D = 3.87}$$

Answer.

**Question # 7:-** For the following distribution of marks calculate Range.

| Marks in percentage | Frequency/No. of students |
|---|---|
| 31 – 40 | 28 |
| 41 – 50 | 31 |
| 51 – 60 | 12 |
| 61 – 70 | 9 |
| 71 – 75 | 5 |
| | |

Sol//

| Class Limit | Class Boundaries | Frequency |
|---|---|---|
| 31 – 40 | 30.5 – 40.5 | 28 |
| 41 – 50 | 40.5 – 50.5 | 31 |
| 51 – 60 | 50.5 – 60.5 | 12 |
| 61 – 70 | 60.5 – 70.5 | 9 |
| 71 – 75 | 70.5 – 75.5 | 5 |



Here $X_{max} = 75.5$ and $X_{min} = 30.5$

$$Range = X_{max} - X_{min}$$

$$Range = 75.5 - 30.5$$

$$\boxed{Range = 45}$$ Answer

# MISCELLANEOUS EXERCISE #6

**Question #1 :-** M.C.Q(s) Doyourself

**Question #2 :-** Write a short answers of the following questions.

**Q#2(i) :-** Define Class Limits?

**Ans:-** The minimum and maximum values defined for a class or group are called class limits.

**Q#2(ii) :-** Define class mark.

**Ans:-** The mid-point of a class interval is also called class mark. It is obtained by dividing the sum of upper and lower limit by 2.



**Q#2(iii) :-** What is cumulative frequency.

**Ans:-** The total of frequency up to an upper class limit or boundary is called the cumulative frequency.

**Q#2(iv) :-** Define a frequency distribution.

**Ans:-** A table showing frequencies against the class intervals is called a frequency distribution table.

Q#2(v):- What is Histogram?

Ans:- A histogram is a graph of adjacent rectangles constructed on XY-plane.

Q#2(vi):- Name two measures of central tendency?

Ans:- (i) Arithmetic mean

(ii) Geometric mean

(iii) Harmonic mean.

Q#2(vii):- Define Arithmetic mean.



Ans:- Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by the number of values.

Q#2(viii):- Write three properties of Arithmetic mean.

Ans:- (i) mean is affected by change in origin.

(ii) Mean is affected by change in scale.

(iii) Sum of deviations of the variable $x$ from its mean is always zero.

**Q#2 (ix):- Define median.**

**Ans:-** Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.

**Q#2(x):- Define mode.**

**Ans:-** The most frequent occurring observation in the data is mode.



**Q#2(xi):- What do you mean by Harmonic mean.**

**Ans:-** Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of $x_1, x_2, x_3, \ldots x_n$, observation.

**Q#2(xii):- Define Geometric mean.**

**Ans:-** Geometric mean of a variable $x$ is the nth positive root of the product of the $x_1, x_2, x_3, \ldots x_n$ observation.

**Q#2(xiii):- What is Range?**

Ans:- Range measures the extent of variation between two extreme observations of a data set.

**Q#2(xiv):- Define standard deviation.**

Ans:- Standard deviation is the positive square root of mean of the squared deviations of $x_i$ $(i = 1, 2, 3, \ldots\ldots n)$ observations from their arithmetic mean.